Matrices and Determinants

Question1

If $A=\begin{bmatrix}-1 & x & -3\\ 2 & 4 & z\\ y & 5 & -6\end{bmatrix}$ **is a symmetric matrix and** $B=\begin{bmatrix}0 & 2 & q\\ p & 0 & -4\\ -3 & r & s\end{bmatrix}$ **is a skew-symmetric matrix, then** $|A|+|B|-|AB|=$

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Options:

A.

$xyz+pqr$

B.

$xyz+q+r$

C.

$\frac{xyz}{pq}$

D.

$xyz+pq+rs$

Answer: B

Solution:

$A=\begin{vmatrix}-1 & x & -3\\ 2 & 4 & z\\ y & 5 & -6\end{vmatrix}$ is a symmetric

So, $A=A^T$

that means, $x=2, y=-3, z=5$

and $B=\begin{bmatrix}0 & 2 & q\\ p & 0 & -4\\ -3 & r & s\end{bmatrix}$ is a skew-matrix

So, $B=-B^T$

that means, $p=-2q=3, r=4, s=0$

Therefore, $|A|=\begin{vmatrix}-1 & 2 & -3\\ 2 & 4 & 5\\ -3 & 5 & -6\end{vmatrix}$

$=-1(-49)-2(3)-3(22)=-23$

and $|B|=\begin{vmatrix}0 & 2 & 3\\ -2 & 0 & -4\\ -3 & 4 & 0\end{vmatrix}$

$=-2(-12)+3(-8)=0$

Now, $|AB|=|A||B|=-23\times 0=-23$

therefore, $|A|+|B|-|AB|=-23$ and from the given options

$xyz+q+r=2\times(-3)\times(5)+3+4=-23$

Hence, $|A|+|B|-|AB|=xyz+q+r$

Question2

If the inverse of $\begin{bmatrix}-x & 14x & 7x\\ 0 & 1 & 0\\ x & -4x & -2x\end{bmatrix}$ **is** $\begin{bmatrix}2 & 0 & 7\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix}$**, then** $\begin{vmatrix}x & x+1 & x+2\\ x+1 & x+2 & x+3\\ x+2 & x+3 & x+4\end{vmatrix}=$

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Options:

A.

$\frac{x}{5}$

B.

$x-5$

C.

$5x-1$

D.

$x+5$

Answer: C

Solution:

$$A=\begin{bmatrix}-x & 14x & 7x\\ 0 & 1 & 0\\ x & -4x & -2x\end{bmatrix}$$

and $B=\begin{bmatrix}2 & 0 & 7\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix}$

If $A^{-1}=B \Rightarrow$ So, $AB=I$

$$\Rightarrow \begin{bmatrix}-x & 14x & 7x\\ 0 & 1 & 0\\ x & -4x & -2x\end{bmatrix}\begin{bmatrix}2 & 0 & 7\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix}=5$$
$$\Rightarrow -2x+7x=1 \Rightarrow x=1/5$$

Now, $\begin{bmatrix}x & x+1 & x+2\\ x+1 & x+2 & x+3\\ x+2 & x+3 & x+4\end{bmatrix}$

$R_2 \rightarrow R_2-R_1$ and $R_3 \rightarrow R_3-R_2$

$$=\begin{bmatrix}x & x+1 & x+2\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

therefore determinant of the given matrix $=0$

Hence, from the given options put $x=1/5$ in $5x-1$ we get, $5\times\frac{1}{5}-1=0$

that means,
$$\begin{vmatrix}x & x+1 & x+2\\ x+1 & x+2 & x+3\\ x+2 & x+3 & x+4\end{vmatrix}=5x-1$$

Question3

If the system of equations $2x+3y-3z=3, x+2y+0z=12x-y+z=\beta$ has infinitely many solutions, then $\frac{\alpha}{\beta}-\frac{\beta}{\alpha}=$

Options:

A.

$\frac{53}{14}$

B.

$\frac{45}{14}$

C.

$-\frac{53}{14}$

D.

$-\frac{45}{14}$

Answer: B

Solution:

$2x + 3y - 3z = 3$

$x + 2y + \alpha z = 1$

and $2x - y + z = \beta$

So, $A = \begin{bmatrix} 2 & 3 & -3 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{bmatrix}$

for infinite many solutions put $|A| = 0$

$$\Rightarrow 2(2+\alpha) - 3(1-2\alpha) - 3(-1-4) = 0$$
$$\Rightarrow 1 + 8\alpha + 15 = 0 \Rightarrow \alpha = -2$$

Now, $D_y = \begin{vmatrix} 2 & 3 & -3 \\ 1 & 1 & -2 \\ 2 & \beta & 1 \end{vmatrix}$

put $D_y = 0$

$$\Rightarrow 2(1+2\beta) - 3(1+4) - 3(\beta-2) = 0$$
$$\Rightarrow \beta - 7 = 0 \Rightarrow \beta = 7$$

Therefore, $\frac{\alpha}{\beta} - \frac{\beta}{\alpha} = \frac{-2}{7} - \frac{7}{(-2)}$

$$= \frac{-4+49}{14} = \frac{45}{14}$$

Question4

If B is the inverse of a third order matrix A and det $B = k$, then $(\text{adj}(\text{adj A}))^{-1} =$

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Options:

A.

kB

B.

$\frac{1}{k}B$

C.

kB^{-1}

D.

$B + kl$

Answer: A

Solution:

We have,

$B = A^{-1}$

and $|B| = k \Rightarrow |A^{-1}| = k$

$\Rightarrow \frac{1}{|A|} = k \Rightarrow |A| = \frac{1}{k}.$

$\because (\text{adj}(\text{adj } A))^{-1} = (|A|^{n-2}A)^{-1}$ and $n = 3$

$= (|A|^1 \cdot A)^{-1} = \left(\frac{1}{k}A\right)^{-1} = kB.$

Question5

If $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and α, β, γ are the roots of the equation represented by $|A - xI| = 0$, then $\alpha^2 + \beta^2 + \gamma^2 =$

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Options:

A.

50

B.

29

C.

17

D.

27

Answer: D

Solution:

$$A=\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$$

and $|A-XI|=0$

$\Rightarrow$ Characterstic equation of matrix A

$\because \operatorname{tr}(A)=2+3+2=7$

Sum of minor of diagonal elements

$$\begin{aligned} &= (4)+(3)+4=11\\ |A| &= 2(4)-2(1)+1(-1)\\ &= 8-2-1=5\\ \because\quad & |A-xI|=0\\ \Rightarrow\quad & x^3-7x^2+11x-5=0\\ \therefore\quad & \alpha+\beta+\gamma=7\\ &\text{and } \alpha\beta+\beta\gamma+\gamma\alpha=11\\ &\text{and } \alpha\beta\gamma=5\\ &\therefore \alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)\end{aligned}$$

$=49-2\times 11=49-22=27.$

Question6

If the values of x, y and z which satisfy the equations $2x-3y+2z+15=0, 3x+y-z+2=0$ and $x-3y-3z+8=0$ simultaneously are α, β and γ respectively, then

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Options:

A.

$\beta+\gamma=\alpha$

B.

$\alpha+\beta=2\gamma$

C.

$2\alpha+\beta=\gamma$

D.

$2\beta+\gamma=2\alpha$

Answer: C

Solution:

Given, $2x-3y+2z=-15$

$3x+y-z=-2$

$x-3y-3z=-8$

$$\therefore \Delta=\begin{vmatrix}2 & -3 & 2\\ 3 & 1 & -1\\ 1 & -3 & -3\end{vmatrix}$$

$$=2(-3-3)+3(-9+1)+2(-9-1)$$

$$=-12-24-20=-56$$

$$\Delta_x=\begin{vmatrix}-15 & -3 & 2\\ -2 & 1 & -1\\ -8 & -3 & -3\end{vmatrix}$$

$$=-15(-3-3)+3(6-8)+2(6+8)$$

$$=90-6+28=112$$

$$\Delta_y=\begin{vmatrix}2 & -15 & 2\\ 3 & -2 & -1\\ 1 & -8 & -3\end{vmatrix}$$

$$=2(6-8)+15(-9+1)+2(-24+2)$$

$$=-4-120-44=-168$$

$$\Delta_z=\begin{vmatrix}2 & -3 & -15\\ 3 & 1 & -2\\ 1 & -3 & -8\end{vmatrix}$$

$$= 2(-8-6) + 3(-24+2) - 15(-9-1)$$
$$= -28 - 66 + 150 = 150 - 94 = 56.$$
$$\alpha = \frac{\Delta_x}{\Delta}, \beta = \frac{\Delta_y}{\Delta} \text{ and } \gamma = \frac{\Delta_z}{\Delta}$$
$$\Rightarrow \alpha = \frac{112}{-56}, \beta = \frac{-168}{-56} \text{ and } \gamma = \frac{56}{-56}$$
$$\Rightarrow \alpha = -2, \beta = 3 \text{ and } \gamma = -1$$
$$\therefore 2\alpha + \beta = \gamma$$

Question7

If a is the determinant of the adjoint of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 3 \end{bmatrix}$ and b is the determinant of the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & -3 & -1 \\ 2 & 1 & -4 \end{bmatrix}$, then $\frac{b+1}{18b} =$

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Options:

A.

a

B.

$10a$

C.

$2 + a$

D.

$2a$

Answer: A

Solution:

Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -3 & -1 \\ 2 & 1 & -4 \end{bmatrix}$

Also, $a=\det(\operatorname{adj}(A))$ and $b=\det\left(B^{-1}\right)$

Now, $a=\det(\operatorname{adj}(A))=(\det(A))^{n-1}$ for an $n\times n$ matrix.

Since A is 3×3 matrix, so $a=(\det(A))^{3-1}=(\det(A))^2$

$$\begin{aligned}\det A &= 1\cdot\begin{vmatrix}2&3\\3&3\end{vmatrix}-1\cdot\begin{vmatrix}1&3\\2&3\end{vmatrix}+2\cdot\begin{vmatrix}1&2\\2&3\end{vmatrix}\\ &=1\cdot(6-9)-1(3-6)+2(3-4)\\ &=-3+3-2=-2\\ a&=(\det(A))^2=(-2)^2=4 \qquad \ldots(i)\end{aligned}$$

Now, $\det(B)=\begin{vmatrix}1&2&3\\4&-3&-1\\2&1&-4\end{vmatrix}$

$$\begin{aligned}&=1\cdot\begin{vmatrix}-3&-1\\1&-4\end{vmatrix}-2\begin{vmatrix}4&-1\\2&-4\end{vmatrix}+3\cdot\begin{vmatrix}4&-3\\2&1\end{vmatrix}\\ &=(12+1)-2(-16+2)+3(4+6)\\ &=13+28+30=71\end{aligned}$$

So, $\quad b=\det\left(B^{-1}\right)=\dfrac{1}{\det(B)}=\dfrac{1}{71}$

$$b=\frac{1}{71}$$

$$\Rightarrow \quad b+1=\frac{1+71}{71}=\frac{72}{71}$$

Then, $\dfrac{b+1}{18b}=\dfrac{72/71}{18\times\frac{1}{71}}$

$$=\frac{72}{71}\times\frac{71}{18}=\frac{72}{18}=4=a \quad \text{[using Eq. (i)]}$$

$$\therefore \frac{b+1}{18b}=a$$

Question8

Consider two systems of 3 linear equations in 3 unknowns $AX=B$ and $CX=D$. If $AX=B$ has unique solution D and $CX=D$ has unique solution B, then the solution of $\left(A-C^{-1}\right)X=0$ is

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Options:

A.

B

B.

D

C.

$B+D$

D.

$B-D$

Answer: B

Solution:

Given, $AX=B$ and $CX=D$ and $AX=B$ has unique solution D and $CX=D$ has unique solution B.

Since, $X=D$ is the unique solution of $AX=B$

$\therefore \quad AD=B \quad \ldots(i)$

And, $X=B$ is the unique solution of $CX=D$,

$\therefore \quad CB=D \Rightarrow B=C^{-1}D \quad \ldots(ii)$

From Eq. (i) and (ii),

$$AD=C^{-1}D$$
$$\left(A-C^{-1}\right)D=0$$

So, the solution of $\left(A-C^{-1}\right)X=0$ is D

Question9

$f(x)$ is an nth degree polynomial satisfying $f(x)=\frac{1}{2}\begin{vmatrix} f(x) & f\left(\frac{1}{x}\right)-f(x) \\ 1 & f\left(\frac{1}{x}\right) \end{vmatrix}$. If $f(2)=33$, then the value of $f(3)$ is

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Options:

A.

126

B.

214

C.

244

D.

-124

Answer: C

Solution:

Given,

$f(x)=\frac{1}{2}\begin{vmatrix} f(x) & f\left(\frac{1}{x}\right)-f(x) \\ 1 & f\left(\frac{1}{x}\right) \end{vmatrix}$

and $f(2)=33$

Here, $f(x)=\frac{1}{2}\left[f(x)\cdot f\left(\frac{1}{x}\right)-f\left(\frac{1}{x}\right)+f(x)\right] \quad \ldots(i)$

Let

$f(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n$
$(a_0\neq 0)$

Putting this value of $f(x)$ in Eq. (i),

$$f(x)=\frac{1}{2}\left[f(x)\cdot f\left(\frac{1}{x}\right)-f\left(\frac{1}{x}\right)+f(x)\right]$$
$$\Rightarrow f(x)\cdot f\left(\frac{1}{x}\right)=f\left(\frac{1}{x}\right)+f(x)$$

Hence, $f(x)=x^n+1$ or $-x^n+1$

Now, $f(2)=2^n+1=33$

$\Rightarrow \quad 2^n=32$
$\Rightarrow \quad 2^n=2^5$
$\Rightarrow \quad n=5$

And $f(2)=-2^n+1=33$

$\Rightarrow \quad -2^n=33-1=32=2^5$

which is not possible.

$\therefore f(x)=x^n+1=x^5+1$

So, $f(3)=3^5+1=243+1=244$

Question10

If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a matrix A and det $A = 4$, then the value of α is

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Options:

A.

3

B.

22

C.

11

D.

4

Answer: C

Solution:

$$\because P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$$

$$\because \operatorname{adj}(A) = (\det A) \cdot A^{-1}$$
$$\Rightarrow \det(\operatorname{adj}(A)) = (\det A)^{n-1}$$

So, for $n = 3$

$$\det(\operatorname{adj}(A)) = (\det A)^{3-1}$$
$$= 4^2 = 16 \qquad \ldots(i)$$

$$\text{and } \det(A) = 1(12-12) - \alpha(4-6) + 3(4-6)$$
$$= 2\alpha - 6 \qquad \ldots(ii)$$

Hence, $2\alpha - 6 = 16$

$$\Rightarrow \quad 2\alpha = 22$$
$$\Rightarrow \quad \alpha = 11$$

Question11

If α is a real root of the equation $x^3 + 6x^2 + 5x - 42 = 0$, then the determinant of the matrix

$$\begin{bmatrix} \alpha - 1 & \alpha + 1 & \alpha + 2 \\ \alpha - 2 & \alpha + 3 & \alpha - 3 \\ \alpha + 4 & \alpha - 4 & \alpha + 5 \end{bmatrix} \text{ is}$$

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Options:

A.

90

B.

120

C.

-105

D.

-135

Answer: C

Solution:

We are given the equation $x^3 + 6x^2 + 5x - 42 = 0$.

Let's check if $x = 2$ is a root by plugging it in:

$(2)^3 + 6 \times (2)^2 + 5 \times 2 - 42 = 8 + 24 + 10 - 42 = 42 - 42 = 0$

Since the equation equals zero, $x = 2$ is a real root. So, $\alpha = 2$.

Now, we substitute $\alpha = 2$ into the matrix:

$$\begin{bmatrix} \alpha - 1 & \alpha + 1 & \alpha + 2 \\ \alpha - 2 & \alpha + 3 & \alpha - 3 \\ \alpha + 4 & \alpha - 4 & \alpha + 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 5 & -1 \\ 6 & -2 & 7 \end{bmatrix}$$

Next, let's find the determinant of this matrix:

$$= \begin{vmatrix} 1 & 3 & 4 \\ 0 & 5 & -1 \\ 6 & -2 & 7 \end{vmatrix}$$

$= 1 \times (5 \times 7 - (-1) \times (-2)) - 3 \times (0 \times 7 - (-1) \times 6) + 4 \times (0 \times (-2) - 5 \times 6)$

$= 1 \times (35 - 2) - 3 \times (0 + 6) + 4 \times (0 - 30)$

$= 33 - 18 - 120$

$= -105$

Question12

The rank of the matrix $\begin{bmatrix} 2 & -3 & 4 & 0 \\ 5 & -4 & 2 & 1 \\ 1 & -3 & 5 & -4 \end{bmatrix}$ **is**

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Options:

A.

0

B.

3

C.

2

D.

1

Answer: B

Solution:

Given matrix

$$\begin{bmatrix} 2 & -3 & 4 & 0 \\ 5 & -4 & 2 & 1 \\ 1 & -3 & 5 & -4 \end{bmatrix}$$

$R_2 \to R_2 - \frac{5}{2}R_1$ and $R_3 \to R_3 - \frac{1}{2}R_1$

$$= \begin{bmatrix} 2 & -3 & 4 & 0 \\ 0 & 3.5 & -8 & 1 \\ 0 & -1.5 & 3 & -4 \end{bmatrix}$$

$R_3 \to R_3 + \frac{1.5}{3.5}R_2 = R_3 + \frac{3}{7}R_2$

$$= \begin{bmatrix} 2 & -3 & 4 & 0 \\ 0 & 3.5 & -8 & 1 \\ 0 & 0 & -3/7 & 25/7 \end{bmatrix}$$

$\because$ All 3 rows are non-zero

Hence, rank of the matrix $= 3$

Question13

$A = \begin{bmatrix} 0 & k & k \\ k & -4 & -6 \\ k & -3 & -5 \end{bmatrix}$ is a singular matrix for

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Options:

A.

$k = 2$ only

B.

$k = \pm 2$ only

C.

no real value of k

D.

all real values of k

Answer: D

Solution:

Given, $A=\begin{bmatrix}0 & k & k \\ k & -4 & -6 \\ k & -3 & -5\end{bmatrix}$

$\because A$ is an singular matrix.

$$\begin{aligned} \therefore \quad & |A|=0 \\ \Rightarrow \quad & \begin{vmatrix}0 & k & k \\ k & -4 & -6 \\ k & -3 & -5\end{vmatrix}=0 \\ \Rightarrow \quad & 0-k(-5k+6k)+k(-3k+4k)=0 \end{aligned}$$

$\Rightarrow -k^2+k^2=0$ which is true for all real values of k.

$\therefore k \in R$

Question14

If $A=\begin{bmatrix}1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6\end{bmatrix}$ and the rank of A is 2 , then the value of x is equal to

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Options:

A.

1

B.

0

C.

-3

D.

3

Answer: C

Solution:

Given, matrix $A = \begin{bmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{bmatrix}$ and the rank of $A = 2$

$= 1(6 - 28) - 2(-24 - 14) + x(16 + 2)$
$|A| = 54 + 18x$

Since, the rank of A is 2 then the determinant of A must be zero.

So, we set the determinant equal to zero and solve for x.

i.e. $54 + 18x = 0 \Rightarrow x = -3$

Question15

$$\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 1 & \frac{1}{3} \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} \frac{1}{2} & \frac{1}{9} \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} \frac{1}{4} & \frac{1}{27} \\ 3 & 1 \end{vmatrix} + \ldots \infty =$$

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Options:

A.

0

B.

1/2

C.

$-1/2$

D.

-1

Answer: C

Solution:

$$\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 1 & \frac{1}{3} \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} \frac{1}{2} & \frac{1}{9} \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} \frac{1}{4} & \frac{1}{27} \\ 3 & 1 \end{vmatrix} + \ldots + \infty$$
$$= (2 - 3) + \left(1 - 3 \times \frac{1}{3}\right) + \left(\frac{1}{2} - 3 \times \frac{1}{9}\right) + \left(\frac{1}{4} - 3 \times \frac{1}{27}\right) + \left(\frac{1}{8} - 3 \times \frac{1}{81}\right) + \ldots + \infty$$
$$= -1 + 0 + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{2^3} - \frac{1}{3^3}\right) + \ldots + \infty$$

$$= -1 + \left[\left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \ldots \infty\right) - \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \ldots + \infty\right)\right]$$

$$= -1 + \left(\frac{\frac{1}{2}}{1 - \frac{1}{2}}\right) - \left(\frac{\frac{1}{3}}{1 - \frac{1}{3}}\right)$$

(Using sum of infinite GP formula)

$$= -1 + \frac{\frac{1}{2}}{\frac{1}{2}} - \frac{\frac{1}{3}}{\frac{2}{3}} = -1 + 1 - \frac{1}{2} = -\frac{1}{2}$$

Question16

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{bmatrix}$ and $|\text{adj}(\text{adj } A)|(\text{adj } A)^{-1} = kA$, then $k =$

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Options:

A.

1296

B.

216

C.

36

D.

432

Answer: B

Solution:

$$|A| = 1(18-5) - 2(6-10) + 3(1-6)$$
$$= 13 + 8 - 15 = 6$$
$$|\operatorname{adj}(\operatorname{adj} A)| = |A|^4 = 6^4$$
$$\Rightarrow (\operatorname{adj} A)^{-1} = \frac{1}{6} A$$
$$\Rightarrow |\operatorname{adj}(\operatorname{adj} A)| (\operatorname{adj} A)^{-1} = kA$$
$$\Rightarrow 6^4 \cdot \frac{1}{6} A = kA$$
$$\therefore k = 216$$

Question17

If the values $x = \alpha, y = \beta, z = \gamma$ satisfy all the 3 equations $x + 2y + 3z = 4, 3x + y + z = 3$ and $x + 3y + 3z = 2$, then $3\alpha + \gamma =$

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Options:

A.

β

B.

2β

C.

$1 - 2\beta$

D.

$2\beta + 1$

Answer: C

Solution:

$\alpha + 2\beta + 3\gamma = 4 \qquad \ldots(i)$

$3\alpha + \beta + \gamma = 3 \qquad \ldots(ii)$
$\alpha + 3\beta + 3\gamma = 2 \qquad \ldots(iii)$

From Eqs. (ii), $3\alpha + \gamma = 3 - \beta \qquad \ldots(iv)$

Substracting Eq. (iii) from Eq. (i)

$$\begin{array}{l} \alpha+2\beta+3\gamma=4 \\ \alpha+3\beta+3\gamma=2 \\ \underline{-\quad-\quad-\quad-} \\ -\beta=2 \Rightarrow \beta=-2 \end{array}$$

Bu Eq. (iv)

$$\begin{aligned} & 3\alpha+\gamma=3-(-2)=5 \\ \Rightarrow & 3\alpha+\gamma=1-2\beta \end{aligned}$$

Question18

The number of solutions of the system of equations $2x+y-z=7, x-3y+2z=1, x+4y-3z=5$ is

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Options:

A.

1

B.

0

C.

Infinite

D.

2

Answer: B

Solution:

$2x+y-z=7, x-3y+2z=1,$
$x+4y-3z=5$
$$A=\begin{bmatrix}2 & 1 & -1\\ 1 & -3 & 2\\ 1 & 4 & -3\end{bmatrix}$$
$$\Rightarrow |A|=\begin{vmatrix}2 & 1 & -1\\ 1 & -3 & 2\\ 1 & 4 & -3\end{vmatrix}$$
$=2(9-8)-1(-3-2)-1(4+3)$
$=2+5-7=0$
$\Rightarrow$ Number of solution = 0

Question19

The value of p and q is that system of equations $2x+py+6z=8, x+2y+qz=5$ and $x+y+3z=4$ may have no solution are

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Options:

A.

$p\neq 2, q=3$

B.

$p\neq 2, q\neq 3$

C.

$p=2, q=\frac{15}{4}$

D.

$p=2, q=3$

Answer: A

Solution:

$2x+py+6z=8, x+2y+qz=5$ and $x+y+3z=4$ may have no solution.

$$x=\frac{\Delta x}{\Delta}, y=\frac{\Delta y}{\Delta}, z=\frac{\Delta z}{\Delta}$$

$$\Delta=\left|\begin{array}{lll}2 & p & 6 \\ 1 & 2 & q \\ 1 & 1 & 3\end{array}\right|=(p-2)(q-3)$$

$$\Delta_x=\left|\begin{array}{lll}8 & p & 6 \\ 5 & 2 & q \\ 4 & 1 & 3\end{array}\right|=(15-4q)(2-p)$$

$$\Delta_y=\left|\begin{array}{lll}2 & 8 & 6 \\ 1 & 5 & q \\ 1 & 4 & 3\end{array}\right|=0$$

$$\Delta_z=\left|\begin{array}{lll}2 & p & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4\end{array}\right|=p-2$$

For no solution

$(p-2)(q-3)=0$

$p-2 \neq 0$

$q=3$

$p \neq 2$

Question20

A is the set of all matrices of order 3 with entries 0 or 1 only. B is the subset of A consisting of all matrices with determinant value 1 . If C is the subset of A consisting of all matrices with determinant value -1 , then

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Options:

A.

$A=B \cup C$

B.

C is empty

C.

B and C contain the same number of elements

D.

B has twice as many elements as C

Answer: C

Solution:

A is det of order 3 with entries 0 or 1 .

$B \subseteq A$ and $|B|=1$ and $C \subset A$ of and $|C|=-1$

$\therefore B$ and C has same number of elements.

Question21

Consider the matrices $A=\begin{bmatrix} x & y & 0 \\ -3 & 1 & 2 \\ 1 & -2 & z \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -2 & -2 \\ 2 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

If the cofactors of the elements $z, 1$ in 3rd row and x of A are $9, 4, 3$, respectively then $AB=$

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Options:

A.

$$\begin{bmatrix} -7 & -4 & -8 \\ -1 & 8 & 7 \\ 3 & -3 & -4 \end{bmatrix}$$

B.

$$\begin{bmatrix} 7 & -6 & -8 \\ -5 & 4 & -5 \\ -5 & -3 & -4 \end{bmatrix}$$

C.

$$\begin{bmatrix} 7 & -6 & -4 \\ 3 & 8 & 7 \\ -5 & -3 & -4 \end{bmatrix}$$

D.

$$\begin{bmatrix} 7 & -6 & 8 \\ -1 & 8 & -5 \\ 3 & -3 & -4 \end{bmatrix}$$

Answer: C

Solution:

(c) $A=\begin{bmatrix} x & y & 0 \\ -3 & 1 & 2 \\ 1 & -2 & z \end{bmatrix}$ and

$B=\begin{bmatrix} 1 & -2 & -2 \\ 2 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

Cofactor of $z=9$, cofactor of 1 in 3^{rd} row of $A=4$

$$\begin{aligned} & x+3y=9 && \ldots(i) \\ \Rightarrow \quad & x=3 \\ & 2y=4 && \ldots(ii) \\ \Rightarrow \quad & y=2 \end{aligned}$$

Cofactor of $x=3$

$z+4=3 \Rightarrow z=-1 \quad \ldots(iii)$

$$\therefore AB=\begin{bmatrix} 3 & 2 & 0 \\ -3 & 1 & 2 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -2 \\ 2 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 7 & -6 & -4 \\ 3 & 8 & 7 \\ -5 & -3 & -4 \end{bmatrix}$$

Question22

If $A=\begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix}$, then $A+2A^{-1}=$

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Options:

A.

$\begin{bmatrix} 1 & 4 & 0 \\ 4 & -5 & -4 \\ 0 & -2 & -7 \end{bmatrix}$

B.

$$\begin{bmatrix} 0 & 2 & 2 \\ 2 & -4 & -6 \\ 2 & -3 & -5 \end{bmatrix}$$

C.

$$\begin{bmatrix} 0 & 2 & 1 \\ 2 & -4 & -3 \\ 2 & -6 & -5 \end{bmatrix}$$

D.

$$\begin{bmatrix} 1 & 4 & -1 \\ 4 & -5 & -1 \\ 1 & -5 & -7 \end{bmatrix}$$

Answer: A

Solution:

$$\text{We have, } A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix}$$

$$|A| = |(2-2) - 2(-4+2) - 2(2-1)| = 4 - 2 = 2$$

$$\text{adj}(A) = \begin{bmatrix} 0 & 2 & 2 \\ 2 & -4 & -6 \\ 1 & -3 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{2}\begin{bmatrix} 0 & 2 & 2 \\ 2 & -4 & -6 \\ 1 & -3 & -5 \end{bmatrix}$$

$$\text{Then, } A + 2A^{-1} = \begin{bmatrix} 1 & 4 & 0 \\ 4 & -5 & -4 \\ 0 & -2 & -7 \end{bmatrix}$$

Question23

If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ l & m & n \end{bmatrix}$ **is a matrix such that** $|A| > 0$ **and** $\text{adj}(A) = \begin{bmatrix} 0 & 4 & -6 \\ 10 & 8 & 0 \\ 2 & 4 & -4 \end{bmatrix}$**, then** $\frac{cd}{fb} + \frac{ln}{em} =$

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Options:

A.

$2a$

B.

$a+m$

C.

$a+b$

D.

a

Answer: B

Solution:

We have, $A=\begin{bmatrix} a & b & c \\ d & e & f \\ l & m & n \end{bmatrix}$ and $\operatorname{adj}(A)=\begin{bmatrix} 0 & 4 & -6 \\ 10 & 8 & 0 \\ 2 & 4 & -4 \end{bmatrix} |\operatorname{adj} A|=|A|^2 \Rightarrow |A|^2=16 \Rightarrow |A|=4$

We know that $\operatorname{adj}(\operatorname{adj} A)=|A|^{n-2}A=|A|^{3-2}A=|A|A$

$$\Rightarrow \quad A=\frac{1}{|A|}\operatorname{adj}(\operatorname{adj} A)$$

$$\text{Here, } \operatorname{adj}(\operatorname{adj} A)=\begin{bmatrix} -32 & -8 & 48 \\ 40 & 12 & -60 \\ 24 & 8 & -40 \end{bmatrix}$$

$$\therefore \quad A=\frac{1}{4}\begin{bmatrix} -32 & -8 & 48 \\ 40 & 12 & -60 \\ 24 & 8 & -40 \end{bmatrix}=\begin{bmatrix} -8 & -2 & 12 \\ 10 & 3 & -15 \\ 6 & 2 & -10 \end{bmatrix}$$

$\Rightarrow a=-8, b=-2, c=12, d=10, e=3, f=-15, l=6, m=2,$
$n=-10$

$$\therefore \quad \frac{cd}{fb}+\frac{\ln}{em}=\frac{12\times 10}{(-15)\times(-2)}+\frac{6\times(-10)}{3\times 2}$$
$$=\frac{120}{30}-\frac{60}{6}=4-10=-6$$

Also, (a) $2a=-16$

(b) $a+m=-8+2=-6$

(c) $a+b=-8-2=-10$

(d) $a=-8$

Question24

In solving a system of linear equations $AX = B$ by Cramer's rule, in the usual notation, if $\Delta_1 = \begin{vmatrix} -11 & 1 & -7 \\ -4 & 1 & -2 \\ 5 & 1 & 1 \end{vmatrix}$ and $\Delta_3 = \begin{vmatrix} 4 & 1 & -11 \\ 1 & 1 & -4 \\ 4 & 1 & 5 \end{vmatrix}$, then $X =$

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Options:

A.

$$\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

B.

$$\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

C.

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

D.

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Answer: C

Solution:

$$Given, \Delta_1 = \begin{vmatrix} -11 & 1 & -7 \\ -4 & 1 & -2 \\ 5 & 1 & 1 \end{vmatrix}$$

$$\because \Delta\Delta_1 = -11(3) - 1(6) - 7(-9) = -33 - 6 + 63 = 24$$

$$\Delta_3 = \begin{vmatrix} 4 & 1 & -11 \\ 1 & 1 & -4 \\ 4 & 1 & 5 \end{vmatrix} = 4(9) - 1(21) - 11(-3) = 48$$

$$\Delta = \begin{vmatrix} 4 & 1 & -7 \\ 1 & 1 & -2 \\ 4 & 1 & 1 \end{vmatrix} = 4(3) - 1(9) - 7(-3) = 24$$

By inspection

$$\Delta_2 = \begin{vmatrix} 4 & -11 & -7 \\ 1 & -4 & -2 \\ 4 & 5 & 1 \end{vmatrix} = 4(6) + 11(9) - 7(21) = -24$$

$$x = \frac{\Delta_1}{\Delta} = \frac{24}{24} = 1, y = \frac{\Delta_2}{\Delta} = \frac{-24}{24} = -1 \text{ and } z = \frac{\Delta_3}{\Delta} = \frac{48}{24} = 2$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Question25

If A and B are both 3×3 matrices, then which of the following statements are true?

(i) $AB = 0 \Rightarrow A = 0$ **or** $B = 0$

(ii) $AB = I_3 \Rightarrow A^{-1} = B$

(iii) $(A - B)^2 = A^2 - 2AB + B^2$

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Options:

A.

(i) is false and (ii), (iii) are true

B.

(ii) is true (i), (iii) are false

C.

(i) and (ii) are true, (iii) is false

D.

All are true

Answer: B

Solution:

(i) $AB = 0 \Rightarrow A = 0$ or $B = 0$

Which is false.

(ii) If $AB = BA = I_3 \Rightarrow A^{-1} = B$

$\because$ It is true.

$$\begin{aligned}(iii)\ (A-B)^2 &= (A-B)(A-B)\\ &= A^2 - AB - BA + B^2\end{aligned}$$

$\Rightarrow A^2 - 2AB + B^2$ is true only when

$AB = BA$

$\because$ It is false.

Question26

$A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & -3 \end{bmatrix}$ is the given matrix and A^T represents the transpose of A, then $AA^T - A - A^T =$

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Options:

A.

$$\begin{bmatrix} 4 & 8 & 12 \\ 8 & 16 & -28 \\ 12 & -28 & 47 \end{bmatrix}$$

B.

$$\begin{bmatrix} 4 & -8 & 12 \\ -8 & 16 & -28 \\ 12 & -28 & 47 \end{bmatrix}$$

C.

$$\begin{bmatrix} 4 & -8 & 12 \\ -8 & 16 & 28 \\ 12 & 28 & 47 \end{bmatrix}$$

D.

$$\begin{bmatrix} 4 & -8 & -12 \\ -8 & 16 & -28 \\ -12 & -28 & 47 \end{bmatrix}$$

Answer: B

Solution:

Given, $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & -3 \\ 4 & -4 & 5 \end{bmatrix}$ then

$A^T = \begin{bmatrix} 1 & -2 & 4 \\ -1 & 3 & -4 \\ 2 & -3 & 5 \end{bmatrix}$

Now, $AA^T = \begin{bmatrix} 6 & -11 & 18 \\ -11 & 22 & -35 \\ 18 & -35 & 57 \end{bmatrix}$

Then, $AA^T - A - A^T = \begin{bmatrix} 4 & -8 & 12 \\ -8 & 16 & -28 \\ 12 & -28 & 47 \end{bmatrix}$

Question27

If $A = \begin{bmatrix} x & 2 & 1 \\ -2 & y & 0 \\ 2 & 0 & -1 \end{bmatrix}$, x and y are non-zero numbers, trace of $A = 0$ and determinant of $A = -6$, then the minor of the elements 1 of A is

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Options:

A.

-4

B.

4

C.

2

D.

-2

Answer: A

Solution:

We have, $A=\begin{bmatrix} x & 2 & 1 \\ -2 & y & 0 \\ 2 & 0 & -1 \end{bmatrix}$

$$\begin{aligned} & trace\ of\ (A)=x+y-1=0 \quad \ldots (i) \\ & \det A=-2y-xy-4=-6 \\ & 2y+xy=2 \quad \ldots (ii) \end{aligned}$$

Solving Eqs. (i) and (ii), we get

$$\begin{aligned} & 2y+y(1-y)=2 \\ \Rightarrow \quad & y^2-3y+2=0 \end{aligned}$$

$y=1$ and 2

and $x=0$ and -1

Then, the minor of the element 1 is

$$\begin{vmatrix} -2 & y \\ 2 & 0 \end{vmatrix}$$
$$=-2y=-2(2)=-4$$

Question28

4. If $A=\begin{bmatrix} 83 & 74 & 41 \\ 93 & 96 & 31 \\ 24 & 15 & 79 \end{bmatrix}$, then $\det\left(A-A^T\right)$ is equal to

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Options:

A. 0

B. -7851

C. 2442

D. 1

Answer: A

Solution:

$$\text{We have, } A=\begin{bmatrix}83 & 74 & 41\\ 93 & 96 & 31\\ 24 & 15 & 79\end{bmatrix}$$

$$A^T=\begin{bmatrix}83 & 93 & 24\\ 74 & 96 & 15\\ 41 & 31 & 79\end{bmatrix}.$$

$$\therefore A-A^T=\begin{bmatrix}83 & 74 & 41\\ 93 & 96 & 31\\ 24 & 15 & 79\end{bmatrix}-\begin{bmatrix}83 & 93 & 24\\ 74 & 96 & 15\\ 41 & 31 & 79\end{bmatrix}$$

$$=\begin{bmatrix}0 & -19 & 17\\ 19 & 0 & 16\\ -17 & -16 & 0\end{bmatrix}$$

$$\therefore \det\left(A-A^T\right)=\begin{vmatrix}0 & -19 & 17\\ 19 & 0 & 16\\ -17 & -16 & 0\end{vmatrix}$$

$$0(0+16\times 16)+19(9\times 0+16\times 17)$$
$$+17(-19\times 16-0)$$
$$=19\times 16\times 17-17\times 19\times 16$$
$$=0$$

Question29

If $\begin{vmatrix}a & 1 & 1\\ 1 & b & 1\\ 1 & 1 & c\end{vmatrix}>0$**, then** $abc>$

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Options:

A. 1

B. -8

C. 8

D. 3

Answer: B

Solution:

We have, $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} > 0$

$= \{(abc - a) - 1(c - 1) + 1(1 - b)\} > 0$

$= (abc - a - c + 1 + 1 - b) > 0$

$= (abc - (a + b + c) + 2) > 0$

$= abc + 2 > (a + b + c)$

$abc + 2 > 3(abc)^{\frac{1}{3}}$

$\left[\because Am > Gm \Rightarrow \frac{a+b+c}{3} > (a+b)^{1/3}\right]$

let $x = (abc)^{1/3} \Rightarrow abc = x^3$

$x^3 + 2 > 3x$

$x^3 - 3x + 2 > 0$

$(x - 1)^2(x + 2) > 0$

$(x + 2) > 0 = x > -2$

1

$= (abc)^{\frac{1}{3}} > -2 = (abc) > -8$

Question30

If the system of equations $a_1x + b_1y + c_1z = 0, a_2x + b_2y + c_2z = 0$ and $a_3x + b_3y + c_3z = 0$ has only trivial solution, then the rank of $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is

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Options:

A. 2

B. 1

C. 3

D. 0

Answer: C

Solution:

The given system of equations are homogeneous and has only trival solution, then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Rank$[A] = n$, where n is the order of square matrix and

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$\therefore$ Rank$[A] = 3$

Question31

$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 0 \\ 4 & 0 & 3 \end{bmatrix}$ and B is a matrix such that $AB = BA$.If AB is not an identity matrix, then the matrix that can be taken as B is

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Options:

A. $\begin{bmatrix} -9 & -3 & 6 \\ -6 & 8 & -4 \\ 12 & -4 & -2 \end{bmatrix}$

B. $\begin{bmatrix} 9 & -3 & 6 \\ -6 & 8 & -4 \\ -12 & -4 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 9 & -3 & -6 \\ -6 & 8 & -4 \\ -12 & 4 & -2 \end{bmatrix}$

D. $\begin{bmatrix} 9 & -3 & -6 \\ -6 & -8 & 4 \\ -12 & 4 & -2 \end{bmatrix}$

Answer: D

Solution:

Given,

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 0 \\ 4 & 0 & 3 \end{bmatrix}$$

and B is a matrix such that $AB = BA$. And AB is not an identity matrix.

Let

$$B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

Now, $AB = BA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 0 \\ 4 & 0 & 3 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 0 \\ 4 & 0 & 3 \end{bmatrix}$$
$$\begin{bmatrix} b_1 + 2c_1 & b_2 + 2c_2 & b_3 + 2c_3 \\ 2a_1 + 3b_1 & 2a_2 + 3b_2 & 2a_3 + 3b_3 \\ 4a_1 + 3c_1 & 4a_2 + 3c_2 & 4a_3 + 3c_3 \end{bmatrix} = \begin{bmatrix} 2a_2 + 4a_3 & a_1 + 3a_2 & 2a_1 + 3a_3 \\ 2b_2 + 4b_3 & b_1 + 3b_2 & 2b_1 + 3b_3 \\ 2c_2 + 4c_3 & c_1 + 3c_2 & 2c_1 + 3c_3 \end{bmatrix}$$

On comparing two matrix and solve for $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2$ and c_3, we get

$a_1 = 9, a_2 = -3$ and $a_3 = -6$
$b_1 = -6, b_2 = -8$ and $b_3 = 4$
$c_1 = -12 c_2 = 4$ and $c_3 = -2$

So, $B = \begin{bmatrix} 9 & -3 & -6 \\ -6 & -8 & 4 \\ -12 & 4 & -2 \end{bmatrix}$

Question32

If α, β and $\gamma (\alpha < \beta < \gamma)$ are the values of x such that $\begin{bmatrix} x-2 & 0 & 1 \\ 1 & x+3 & 2 \\ 2 & 0 & 2x-1 \end{bmatrix}$ is a singular matrix, then $2\alpha + 3\beta + 4\gamma$ is equal to

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Options:

A. 4

B. 0

C. 1

D. 2

Answer: A

Solution:

Let $A = \begin{bmatrix} x-2 & 0 & 1 \\ 1 & x+3 & 2 \\ 2 & 0 & 2x-1 \end{bmatrix}$ is a singular matrix.

So, $|A| = 0$

$$\begin{vmatrix} x-2 & 0 & 1 \\ 1 & x+3 & 2 \\ 2 & 0 & 2x-1 \end{vmatrix} = 0$$

$$\begin{aligned} &\Rightarrow (x-2)[(x+3)(2x-1)-0]-0 \\ &\Rightarrow \quad (x-2)(x+3)(2x-1)-2(x+3)] = 0 \\ &\Rightarrow \quad (x+3)[(x-2)(2x-1)-2] = 0 \\ &\Rightarrow \quad (x+3)\left[2x^2-x-4x+2-2\right] = 0 \\ &\Rightarrow \quad (x+3)\left[2x^2-5x\right] = 0 \\ &\Rightarrow \quad x(x+3)(2x-5) = 0 \\ &\Rightarrow \quad x = -3, 0, 5/2 \end{aligned}$$

So, $\alpha = -3, \beta = 0$ and $\gamma = 5/2$

[given $\alpha < \beta < \gamma$]

Now,

$$\begin{aligned} 2\alpha + 3\beta + 4\gamma &= 2 \times (-3) + 3 \times 0 + 4 \times \frac{5}{2} \\ &= -6 + 10 = 4 \end{aligned}$$

Question33

The system of linear equations $x + 2y + z = -3, 3x + 3y - 2z = -1$ and $2x + 7y + 7z = -4$ has

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Options:

A. infinite number of solutions

B. no solution

C. unique solution

D. finite number of solutions

Answer: B

Solution:

To determine the nature of the solutions for the system of linear equations:

$$\begin{aligned} x+2y+z&=-3,\\ 3x+3y-2z&=-1,\\ 2x+7y+7z&=-4, \end{aligned}$$

we begin by calculating the determinant Δ of the coefficient matrix:

$$\Delta=\begin{vmatrix}1 & 2 & 1\\ 3 & 3 & -2\\ 2 & 7 & 7\end{vmatrix}$$

Calculating this determinant:

$\Delta=1(21+14)-2(21+4)+1(21-6)$

$=1\times 35-2\times 25+1\times 15$

$=35-50+15=0$

Since $\Delta=0$, the system of equations is either inconsistent (no solution) or dependent (infinitely many solutions). To resolve this, we check Δ_x, the determinant of the matrix when the first column is replaced by the constants from the right side of the equations:

$$\Delta_x=\begin{vmatrix}-3 & 2 & 1\\ -1 & 3 & -2\\ -4 & 7 & 7\end{vmatrix}$$

Calculating Δ_x:

$\Delta_x=(-3)(35)-2(-15)+1(5)$

$=-105+30+5=-70$

Since $\Delta=0$ and $\Delta_x\neq 0$, the system has no solution. Hence, the equations are inconsistent.

Question34

If the set of equations $x+2y+3z=6, x+3y+5z=9,$ $2x+5y+az=b$ has unique solution, then

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Options:

A. $a=8, b=15$

B. $a \neq 8, b \in R$

C. $a=8, b \neq 15$

D. $a \neq 15, b=8$

Answer: B

Solution:

To determine the condition for a and b so, that the set of equations

$x+2y+3z=6$
$x+3y+5z=9$

$2x+5y+az=b$ has a unique solution. We need to ensure that the coefficient matrix has a non zero determinant.

Let $A=\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 5\\ 2 & 5 & a\end{pmatrix}$

Then, $\det(A)$

$$\begin{aligned}&=1(3a-25)-2(\cdot a-10)+3(5-6)\\&=1(3a-25)-2(a-10)+3(-1)\\&=3a-25-2a+20-3\\&=a-8\end{aligned}$$

For system to have a unique solution. $\det(A)$ must be non-zero.

i.e. $a-8 \neq 0$

Thus, $a \neq 8$

So, b can be other number which should be a non-zero real number and therefore b must be a real number.

Thus, $a \neq 8, b \in R$.

Question35

If P and Q are two 3×3 matrices such that $|PQ|=1$ and $|P|=9$, then the determinant of adjoint of the matrix $P.\operatorname{adj} 3Q$ is

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Options:

A. 9^4

B. $\frac{1}{9^4}$

C. 9^2

D. $\frac{1}{9^2}$

Answer: A

Solution:

Given, $|PQ| = 1$ and $|P| = 9$.

For any $n \times n$ matrix A, the determinant of $\operatorname{adj}(A)$ is $|A|^{n-1}$, where n is the dimension of the square matrix.

If $B = kA$, where k is a scalar, then $|B| = k^n|A|$ for an $n \times n$ matrix A.

Also, $|PQ| = |P||Q|$.

Utilizing these properties, for a 3×3 matrix we have:

$|3Q| = 3^3|Q| = 27|Q|$.

Now, considering the matrix $\operatorname{adj}(3Q)$:

$\operatorname{Det}(\operatorname{adj}(3Q)) = (27|Q|)^2$.

Since $|PQ| = 1$ and $|P| = 9$, we have:

$$1 = 9 \cdot |Q|,$$
$$|Q| = \frac{1}{9}.$$

Finally, compute $\det(\operatorname{adj}(3Q))$:

$$\begin{aligned}\det(\operatorname{adj}(3Q)) &= (27|Q|)^2 \\ &= (27 \times \frac{1}{9})^2 \\ &= 3^2 \\ &= 9.\end{aligned}$$

For $\operatorname{adj}(P \cdot \operatorname{adj}(3Q))$:

$$\begin{aligned}\operatorname{adj}(P \cdot \operatorname{adj}(3Q)) &= (9 \times 9)^{3-1} \\ &= (9^2)^2 \\ &= 9^4.\end{aligned}$$

Question36

If $A = \begin{bmatrix} a & 1 & 2 \\ 1 & 2 & b \\ c & 1 & 3 \end{bmatrix}$ **and** $\operatorname{adj} A = \begin{bmatrix} 7 & -1 & -5 \\ -3 & 9 & 5 \\ 1 & -3 & 5 \end{bmatrix}$**, then** $a^2 + b^2 + c^2 =$

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Options:

A. 10

B. 14

C. 11

D. 29

Answer: A

Solution:

We know that for a square matrix the product $A \cdot \operatorname{adj}(A) = \det(A)$. I where I is the identity matrix of the same order as A.

If A is a 3×3 matrix, then the product should be : $A \cdot \operatorname{adj}(A) = \det(A) \cdot I_3$

Given matrix

$$A = \begin{bmatrix} a & 1 & 2 \\ 1 & 2 & b \\ c & 1 & 3 \end{bmatrix}$$
$$\operatorname{adj}(A) = \begin{bmatrix} 7 & -1 & -5 \\ -3 & 9 & 5 \\ 1 & -3 & 5 \end{bmatrix}$$

Let's multiply A with adj (A)

$$A \cdot \operatorname{adj}(A) = \begin{bmatrix} a & 1 & 2 \\ 1 & 2 & b \\ c & 1 & 3 \end{bmatrix} \begin{bmatrix} 7 & -1 & -5 \\ -3 & 9 & 5 \\ 1 & -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 7a - 1 & b + 1 & 7c \\ -a + 3 & -3b + 17 & -c \\ -5a + 15 & 5b + 5 & -5 + 20 \end{bmatrix}$$

Setting these elements equal to identity matrix adjusted by determinant

We get, $-a + 3 = 0$

$$\Rightarrow \quad a = 3, b + 1 = 0 \Rightarrow b = -1$$
$$7c = 0 \Rightarrow c = 0$$

Thus, $a=3, b=-1$ and $c=0$

Hence, $a^2+b^2+c^2$

$=3^2+(-1)^2+0=9+1=10$

Question37

If $3A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$ **and** $AA^T=I$**, then** $\frac{a}{b}+\frac{b}{a}=$

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Options:

A. $\frac{-5}{2}$

B. $\frac{13}{6}$

C. $-\frac{13}{6}$

D. $\frac{5}{2}$

Answer: D

Solution:

Given that $3A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$

$\Rightarrow A=\begin{bmatrix}\frac{1}{3} & \frac{2}{3} & \frac{2}{3}\\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3}\\ \frac{a}{3} & \frac{2}{3} & \frac{b}{3}\end{bmatrix}$

$\therefore A^T=\begin{bmatrix}\frac{1}{3} & \frac{2}{3} & \frac{a}{3}\\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3}\\ \frac{2}{3} & -\frac{2}{3} & \frac{b}{3}\end{bmatrix}$

Now, $AA^T=I\Rightarrow\begin{bmatrix}\frac{1}{3} & \frac{2}{3} & \frac{2}{3}\\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3}\\ \frac{a}{3} & \frac{2}{3} & \frac{b}{3}\end{bmatrix}\begin{bmatrix}\frac{1}{3} & \frac{2}{3} & \frac{a}{3}\\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3}\\ \frac{2}{3} & -\frac{2}{3} & \frac{b}{3}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}\frac{1}{9}+\frac{4}{9}+\frac{4}{9} & \frac{2}{9}+\frac{2}{9}-\frac{4}{9} & \frac{a}{9}+\frac{4}{9}+\frac{2b}{9}\\ \frac{2}{9}+\frac{2}{9}-\frac{4}{9} & \frac{4}{9}+\frac{1}{9}+\frac{4}{9} & \frac{2a}{9}+\frac{2}{9}-\frac{2b}{9}\\ \frac{a}{9}+\frac{4}{9}+\frac{2b}{9} & \frac{2a}{9}+\frac{2}{9}-\frac{2b}{9} & \frac{a^2}{9}+\frac{4}{9}+\frac{b^2}{9}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{a+4+2b}{9} \\ \frac{a+4+2b}{9} & \frac{2a+2-2b}{9} & \frac{\frac{a^2+4+b^2}{9}}{0} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \quad a^2 + b^2 + 4 = 9 \qquad \ldots(i)$

$\Rightarrow \quad a^2 + b^2 = 5$

and $2a + 2 - 2b = 0$

$\Rightarrow \quad b = a + 1 \qquad \ldots(ii)$

and $a + 4 + 2b = 0$

$\Rightarrow \quad a + 2b = -4 \qquad \ldots(iii)$

On solving Eqs. (ii) and (iii), we get

$a + 2b = -4$

$\Rightarrow \quad a + 2(a+1) = -4$

$\Rightarrow \quad a + 2a + 2 = -4$

$\Rightarrow \quad 3a = -6$

$\Rightarrow \quad a = -2$ and $b = -2 + 1$

$\Rightarrow \quad b = -1$

$$Now, \quad \frac{a}{b} + \frac{b}{a} = \frac{-2}{-1} + \frac{-1}{-2} = 2 + \frac{1}{2} = \frac{5}{2}$$

Question38

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} =$$

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Options:

A. $(a+b+c)^3$

B. $2(a+b+c)^3$

C. $3(a+b+c)^3$

D. $(a+b+c)$

Answer: B

Solution:

We have,

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(b+c+a) & b+c+2a & b \\ 2(c+a+b) & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2$ and $R_2 \rightarrow R_2 - R_3$

$$= 2(a+b+c) \begin{vmatrix} 0 & -(a+b+c) & 0 \\ 0 & a+b+c & -(a+b+c) \\ 1 & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c)^3 \begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c)^3[1(1-0)] = 2(a+b+c)^3$$

Question39

Assertion (A) : If B is a 3×3 matrix and $|B| = 6$, then $|\text{adj}(B)| = 36$

Reason (R) : If B is a square matrix of order n, then $|\text{adj}(B)| = |B|^n$

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Options:

A. Both (A) and (R) are true and (R) is the correct explanation of (A).

B. Both (A) and (R) are true but (R) is not the correct explanation of (A).

C. (A) is true but (R) is false.

D. (A) is false but (R) is true.

Answer: C

Solution:

To clarify:

When we have a matrix B of order 3 with a determinant $|B| = 6$, the formula to find the determinant of its adjugate matrix, $|\text{adj}(B)|$, is given by:

$|\text{adj}(B)| = |B|^{n-1}$

where n is the order of the matrix B.

In this case, since $n = 3$, we have:

$|\text{adj}(B)| = 6^{3-1} = 6^2 = 36$

This confirms that the assertion is true.

However, the reason provided, $|\text{adj}(B)| = |B|^n$, is incorrect because it states that the determinant of the adjugate matrix is raised to the power of n rather than $n-1$. The correct relationship involves raising the determinant of B to the power of $n-1$.

Thus, the assertion is true, but the given reason is false.

Question40

If $A = \begin{vmatrix} 2 & 3 & 4 \\ 1 & k & 2 \\ 4 & 1 & 5 \end{vmatrix}$ is singular matrix, then the quadratic equation having the roots k an $\frac{1}{k}$ is

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Options:

A. $6x^2 + 13x + 6 = 0$

B. $12x^2 - 25x + 12 = 0$

C. $6x^2 - 13x + 6 = 0$

D. $2x^2 - 5x + 2 = 0$

Answer: C

Solution:

Given the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & k & 2 \\ 4 & 1 & 5 \end{bmatrix}$, which is singular, we need to find the quadratic equation with roots k and $\frac{1}{k}$.

Since A is singular, its determinant is zero:

$$|A| = \begin{vmatrix} 2 & 3 & 4 \\ 1 & k & 2 \\ 4 & 1 & 5 \end{vmatrix} = 2(5k-2) - 3(5-8) + 4(1-4k) = 0$$

Expanding this:

$2(5k-2) = 10k - 4$

$-3(5-8) = -3 \cdot 3 = -9$

$4(1-4k) = 4 - 16k$

Putting it all together, we get:

$10k - 4 - 9 + 4 - 16k = 0$

Simplifying:

$-6k + 9 = 0$

Solving for k:

$k = \frac{3}{2}$

Thus, the roots of the quadratic equation are $\frac{3}{2}$ and $\frac{2}{3}$. Using the standard form of a quadratic equation $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ where the roots are $\alpha = \frac{3}{2}$ and $\beta = \frac{2}{3}$, we calculate:

$\alpha + \beta = \frac{3}{2} + \frac{2}{3} = \frac{9}{6} + \frac{4}{6} = \frac{13}{6}$

$\alpha\beta = \frac{3}{2} \cdot \frac{2}{3} = 1$

Substitute these into the quadratic equation:

$x^2 - \left(\frac{13}{6}\right)x + 1 = 0$

To eliminate fractions, multiply through by 6:

$6x^2 - 13x + 6 = 0$

This is the quadratic equation with the specified roots.

Question41

Let A be a 4×4 matrix and P be is adjoint matrix, If $|P| = \left|\frac{A}{2}\right|$ then $\left|A^{-1}\right|$

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Options:

A. $\pm\frac{1}{4}$

B. ± 8

C. ± 2

D. ±4

Answer: D

Solution:

Given that $|P| = \left|\frac{A}{2}\right|$, we start by using the property of the adjoint matrix, where $|\text{adj}(A)| = |A|^{n-1}$ for an $n \times n$ matrix. Here, since A is a 4×4 matrix, $|\text{adj}(A)| = |A|^3$.

Substituting the given condition, we have:

$|A|^3 = \left(\frac{1}{2}\right)^4 |A|$

Simplifying, this becomes:

$|A|^3 = \frac{1}{16}|A|$

Assuming $|A| \neq 0$, divide both sides by $|A|$:

$|A|^2 = \frac{1}{16}$

Taking the square root on both sides gives:

$|A| = \pm\frac{1}{4}$

To find $|A^{-1}|$, we use the property $|A^{-1}| = \frac{1}{|A|}$:

$|A^{-1}| = \pm4$

Therefore, the absolute determinant of the inverse of A is ±4.

Question42

The system $x + 2y + 3z = 4, 4x + 5y + 3z = 5, 3x + 4y + 3z = \lambda$ is consistent and $3\lambda = n + 100$, then $n =$

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Options:

A. -42

B. -86

C. 16

D. -24

Answer: B

Solution:

We have,

$$\begin{aligned} x+2y+3z&=4 \\ 4x+5y+3z&=5 \\ 3x+4y+3z&=\lambda \end{aligned}$$

Write the equation in matrix form $[A_1^1 B]$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 4 \\ 4 & 5 & 3 & 5 \\ 3 & 4 & 3 & \lambda \end{array}\right]$$

$R_2 \rightarrow R_2 - 4R_1$ and $R_3 \rightarrow R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & -9 & -11 \\ 0 & -2 & -6 & \lambda-12 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & -9 & -11 \\ 0 & 0 & 0 & 3\lambda-14 \end{array}\right]$$

$R_3 \leftrightarrow 3R_3 - 2R_2$

For consistency $3\lambda - 14 = 0 \Rightarrow 3\lambda = 14$

Given, $3\lambda = n + 100$

Therefore, $n = 14 - 100$

$n = -86$

So, the value of n is -86 .

Question43

$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$ **is not equal to**

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Options:

A. $\begin{vmatrix} a+1 & b+1 & c+1 \\ a^2+1 & b^2+1 & c^2+1 \\ 1 & 1 & 1 \end{vmatrix}$

B. $\begin{vmatrix} a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \\ 0 & 0 & 1 \end{vmatrix}$

C. $\begin{vmatrix} a(a+1) & b(b+1) & c(c+1) \\ a+1 & b+1 & c+1 \\ -1 & -1 & -1 \end{vmatrix}$

D. $\begin{vmatrix} a+b & b+c & c+a \\ a^2+b^2 & b^2+c^2 & c^2+a^2 \\ 2 & 2 & 2 \end{vmatrix}$

Answer: D

Solution:

Let $\Delta = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix} \quad \ldots \text{(i)}$

Applying, $R_1 \to R_1 + R_3, R_2 \to R_2 + R_3$ we get

$$\Delta = \begin{vmatrix} a+1 & b+1 & 0+1 \\ a^2+1 & b^2+1 & c^2+1 \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$\Delta = \begin{vmatrix} a-b & b-c & c+1 \\ a^2-b^2 & b^2-c^2 & c^2+1 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= 1\left[(a-b)\left(b^2-c^2\right)-(b-c)\left(a^2-b^2\right)\right]$$
$$= 1\left[ab^2-ac^2-b^3+bc^2-a^2b+b^3+a^2c-b^2c\right]$$

$$= ab^2-a^2b-ac^2+a^2c-b^2c+bc^2$$

Thus, option (a) and (b) are correct. Now, option (c),

$$\begin{vmatrix} a(a+1) & b(b+1) & c(c+1) \\ a+1 & b+1 & c+1 \\ -1 & -1 & -1 \end{vmatrix}$$

Applying, $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} a(a+1) & b^2+b-a^2-a & c^2+c-a^2-a \\ a+1 & b-a & c-a \\ -1 & 0 & 0 \end{vmatrix}$$

$$= -1\left[\left(b^2+b-a^2-a\right)(c-a)-\left(c^2+c-a^2-a\right)(b-a)\right]$$

$$= \left(c^2+c-a^2-a\right)(b-a)-\left(b^2+b-a^2-a\right)(c-a)$$

$$= c^2b-ac^2+bc-ac-a^2b+a^3-ab+a^2-b^2c+ab^2-bc+ab+a^2c-a^3+ac-a^2$$

Hence, option (d) is correct required answer.

Question44

Let A, B, C, D and E be $n \times n$ matrices each with non-zero determinant. If $ABCDE = I$, then $C^{-1} =$

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Options:

A. $E^{-1}D^{-1}B^{-1}A^{-1}$

B. $DEAB$

C. $A^{-1}B^{-1}D^{-1}E^{-1}$

D. $ABDE$

Answer: B

Solution:

Given, $ABCDE = I$

$\Rightarrow A^{-1}(ABCDE)E^{-1} = A^{-1}IE^{-1}$

($\because$ Pre multiply by A^{-1} and post multiply by E^{-1}, respectively)

$$\Rightarrow \quad IBCDI = A^{-1}E^{-1}$$
$$\Rightarrow \quad BCD = A^{-1}E^{-1}$$

Now, pre multiply by B^{-1} and post multiply by D^{-1}, we get

$$\Rightarrow \quad B^{-1}(BCD)D^{-1} = B^{-1}A^{-1}E^{-1}D^{-1}$$
$$\Rightarrow \quad B^{-1}BCDD^{-1} = B^{-1}A^{-1}E^{-1}D^{-1}$$
$$\Rightarrow \quad ICI = B^{-1}A^{-1}E^{-1}D^{-1}$$
$$\Rightarrow \quad C = B^{-1}A^{-1}E^{-1}D^{-1}$$
$$\therefore \quad C^{-1} = \left(B^{-1}A^{-1}E^{-1}D^{-1}\right)^{-1}$$
$$= \left(D^{-1}\right)^{-1}\left(E^{-1}\right)^{-1}\left(A^{-1}\right)^{-1}\left(B^{-1}\right)^{-1}$$

(using reversal law)

$$= DEAB$$
$$\therefore \quad C^{-1} = DEAB$$

Question45

If $A = [a_{ij}], 1 \leq i, j \leq n$ with $n \geq 2$ and $a_{ij} = i + j$ is a matrix, then the rank of A is

Options:

A. 0

B. 1

C. 2

D. 4

Answer: C

Solution:

Given, $A=[a_{ij}], 1 \leq i, j \leq n$ with $n \geq 2$ and $a_{ij}=i+j$

$\therefore$ The matrix is

$$A=\begin{bmatrix} 2 & 3 & 4 & \cdots & n+1 \\ 3 & 4 & 5 & \cdots & n+2 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ n+1 & n+2 & n+3 & \cdots & n+n \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 3 & \cdots & n+1 \\ 3 & 4 & \cdots & n+2 \\ 4 & 5 & \cdots & n+3 \\ \vdots & \vdots & \cdots & \vdots \\ n+1 & n+2 & \cdots & 2n \end{bmatrix}$$

Use elementary row operation to subtract 1st row from the i th row for $2 \leq i \leq n$, we get

$$A=\begin{bmatrix} 2 & 3 & 4 & \cdots & n+1 \\ 1 & 1 & 1 & \cdots & 1 \\ 2 & 2 & 2 & \cdots & 2 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ n-1 & n-1 & n-1 & \cdots & n-1 \end{bmatrix}$$

Applying, $R_3 \rightarrow R_3-2R_2$ and $R_4 \rightarrow R_4-3R_2$ and so on

$$A=\begin{bmatrix} 2 & 3 & 4 & \cdots & n+1 \\ 1 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$\therefore$ Rank $A=2$

Question46

If $A=\begin{bmatrix}1 & 0 & 2\\ 2 & 1 & 3\\ 3 & 2 & 4\end{bmatrix}$, then $A^2-5A+6I=$

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Options:

A. $\begin{bmatrix}8 & 4 & 0\\ 3 & 8 & 4\\ 4 & 0 & 12\end{bmatrix}$

B. $\begin{bmatrix}8 & 4 & 0\\ 3 & 6 & 4\\ 4 & 0 & 14\end{bmatrix}$

C. $\begin{bmatrix}8 & 6 & 0\\ 3 & 8 & 4\\ 2 & 0 & 14\end{bmatrix}$

D. $\begin{bmatrix}8 & 4 & 0\\ 3 & 8 & 4\\ 4 & 0 & 14\end{bmatrix}$

Answer: D

Solution:

Given the matrix A:

$$A=\begin{bmatrix}1 & 0 & 2\\ 2 & 1 & 3\\ 3 & 2 & 4\end{bmatrix}$$

We are tasked to find $A^2-5A+6I$.

Calculate A^2:

$$A^2=A\cdot A=\begin{bmatrix}1 & 0 & 2\\ 2 & 1 & 3\\ 3 & 2 & 4\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 2 & 1 & 3\\ 3 & 2 & 4\end{bmatrix}$$

Perform the matrix multiplication:

$$A^2=\begin{bmatrix}1\cdot1+0\cdot2+2\cdot3 & 1\cdot0+0\cdot1+2\cdot2 & 1\cdot2+0\cdot3+2\cdot4\\ 2\cdot1+1\cdot2+3\cdot3 & 2\cdot0+1\cdot1+3\cdot2 & 2\cdot2+1\cdot3+3\cdot4\\ 3\cdot1+2\cdot2+4\cdot3 & 3\cdot0+2\cdot1+4\cdot2 & 3\cdot2+2\cdot3+4\cdot4\end{bmatrix}$$

$$= \begin{bmatrix} 1+0+6 & 0+0+4 & 2+0+8 \\ 2+2+9 & 0+1+6 & 4+3+12 \\ 3+4+12 & 0+2+8 & 6+6+16 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 4 & 10 \\ 13 & 7 & 19 \\ 19 & 10 & 28 \end{bmatrix}$$

Calculate $5A$:

$$5A = 5 \cdot \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 10 \\ 10 & 5 & 15 \\ 15 & 10 & 20 \end{bmatrix}$$

Calculate $6I$:

$$6I = 6 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Combine the results to find $A^2 - 5A + 6I$:

$$A^2 - 5A + 6I = \begin{bmatrix} 7 & 4 & 10 \\ 13 & 7 & 19 \\ 19 & 10 & 28 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 10 \\ 10 & 5 & 15 \\ 15 & 10 & 20 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 7-5+6 & 4-0+0 & 10-10+0 \\ 13-10+0 & 7-5+6 & 19-15+0 \\ 19-15+0 & 10-10+0 & 28-20+6 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 & 0 \\ 3 & 8 & 4 \\ 4 & 0 & 14 \end{bmatrix}$$

Thus, the resulting matrix is:

$$\begin{bmatrix} 8 & 4 & 0 \\ 3 & 8 & 4 \\ 4 & 0 & 14 \end{bmatrix}$$

Question47

Sum of the positive roots of the equation $\begin{vmatrix} x^2+2x & x+2 & 1 \\ 2x+1 & x-1 & 1 \\ x+2 & -1 & 1 \end{vmatrix} = 0$ is

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Options:

A. $\frac{1+\sqrt{13}}{2}$

B. 1

C. $\frac{\sqrt{13}-1}{2}$

D. 3

Answer: A

Solution:

We have, $\begin{vmatrix} x^2+2x & x+2 & 1 \\ 2x+1 & x-1 & 1 \\ x+2 & -1 & 1 \end{vmatrix} = 0$

$$\begin{aligned}
&\Rightarrow \left(x^2+2x\right)\begin{vmatrix} x-1 & 1 \\ -1 & 1 \end{vmatrix} - (x+2)\begin{vmatrix} 2x+1 & 1 \\ x+2 & 1 \end{vmatrix} + 1 \\
&\Rightarrow \left(x^2+2x\right)(x-1+1) - (x+2)(2x+1-x-2) \\
&\quad +1\left(-2x-1-x^2-x+2\right) = 0 \\
&\Rightarrow x\left(x^2+2x\right) - (x+2)(x-1) + 1 \\
&\quad \left(-x^2-3x+1\right) = 0 \\
&\Rightarrow x^3+2x^2-x^2-x+2-x^2-3x+1 = 0 \\
&\Rightarrow x^3-4x+3 = 0
\end{aligned}$$

On putting, $x = 1$, we get

$(1)^3 - 4 \times 1 + 3 = 0$

Thus, $x = 1$ is a root of the given equation.

Thus, $(x-1)\left(x^2+x-3\right) = 0$

Solve $x^2+x-3 = 0$

$$\begin{aligned}
x &= \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-3)}}{2 \times 1} \\
&= \frac{-1 \pm \sqrt{1+12}}{2} = \frac{-1 \pm \sqrt{13}}{2}
\end{aligned}$$

Hence, positive roots are 1 and $\frac{-1+\sqrt{13}}{2}$

Sum of positive roots $= 1 + \left(\frac{-1+\sqrt{13}}{2}\right)$

$= \frac{1+\sqrt{13}}{2}$

Question48

If the solution of the system of simultaneous linear equations $x+y-z=6, 3x+2y-z=5$ and $2x-y-2z+3=0$ is $x=\alpha, y=\beta, z=y$, then $\alpha+\beta=$

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Options:

A. -7

B. 2

C. 1

D. -2

Answer: B

Solution:

Given system of equation

$$\begin{aligned} x+y-z&=6 \\ 3x+2y-z&=5 \\ 2x-y-2z&=-3 \end{aligned}$$

We can represent this as

$A \cdot X = B$

Where

$$A=\begin{bmatrix}1 & 1 & -1\\ 3 & 2 & -1\\ 2 & -1 & -2\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}6\\ 5\\ -3\end{bmatrix}$$

$$\text{Now, } [A/B]=\begin{bmatrix}1 & 1 & -1\\ 3 & 2 & -1\\ 2 & -1 & -2\end{bmatrix}\begin{bmatrix}6\\ 5\\ -3\end{bmatrix}$$

$$R_2 \rightarrow R_2-3R_1=\begin{bmatrix}1 & 1 & -1\\ 0 & -1 & 2\\ 2 & -1 & -2\end{bmatrix}\begin{bmatrix}6\\ -13\\ -3\end{bmatrix}$$

$$R_3 \rightarrow R_3-2R_1=\begin{bmatrix}1 & 1 & -1\\ 0 & -1 & 2\\ 0 & -3 & 0\end{bmatrix}\begin{bmatrix}6\\ -13\\ -15\end{bmatrix}$$

$$R_2 \rightarrow -R_2=\begin{bmatrix}1 & 1 & -1\\ 0 & 1 & -2\\ 0 & -3 & 0\end{bmatrix}\begin{bmatrix}6\\ 13\\ -15\end{bmatrix}$$

$$R_3 \rightarrow R_3+3R_2=\begin{bmatrix}1 & 1 & -1\\ 0 & 1 & -2\\ 0 & 0 & -6\end{bmatrix}\begin{bmatrix}6\\ 13\\ 24\end{bmatrix}$$

$$R_3 \rightarrow \frac{R_3}{-6}$$

$$=\begin{bmatrix}1 & 1 & -1\\ 0 & 1 & -2\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}6\\ 13\\ -4\end{bmatrix}$$

$$R_2 \to R_2 + 2R_3 = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ -4 \end{bmatrix}$$

Now, we get

$$\begin{aligned} x + y - z &= 6 \\ y &= 5 \\ z &= -4 \\ x + 5 + 4 &= 6 \\ x + 9 &= 6 \\ x &= -3 \end{aligned}$$

Thus, $x = \alpha = -3$

$y = \beta = 5$
$z = \gamma = -4$

then, $\alpha + \beta = -3 + 5 = 2$

Question49

$$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} =$$

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Options:

A. $(a-b)(b-c)(c-a)(a+b+c)$

B. $(a-b)(b-c)(c-a)$

C. $(a-b)(b-c)(a-c)(ab+bc+ca)$

D. $(a-b)(b-c)(c-a)(ab+bc+ca)$

Answer: D

Solution:

We have, $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a^2 & b^2 - a^2 & c^2 - a^2 \\ a^3 & b^3 - a^3 & c^3 - a^3 \end{vmatrix}$$

$$= (b-a)(c-a)$$

$$\begin{vmatrix} 1 & 0 & 0 \\ a^2 & (b+a) & (c+a) \\ a^3 & (b^2 + a^2 + ab) & (c^2 + a^2 + ac) \end{vmatrix}$$

$$= (b-a)(c-a)$$

$$\begin{vmatrix} b+a & c+a \\ b^2 + a^2 + ab & c^2 + a^2 + ac \end{vmatrix}$$

$$= \left[(b-a)(c-a)\left[bc^2 + a^2b + abc + ac^2 + a^3\right.\right.$$

$$\left.+a^2c - b^2c - a^3 - a^2b - a^2c - abc - ab^2\right]$$

$$= (b-a)(c-a)\left[bc^2 + ac^2 - b^2c - ab^2\right]$$

$$= (b-a)(c-a)[bc(c-b)$$

$$+ a(c-b)(c+b)]$$

$$= (a-b)(b-c)(c-a)(ab+bc+ca)$$

Question50

If $A = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}$ and $\alpha A^2 + \beta A = 2I$ for some $\alpha, \beta \in R$, then $\alpha + \beta =$

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Options:

A. 7

B. 10

C. 12

D. 5

Answer: B

Solution:

To solve the given equation $\alpha A^2 + \beta A = 2I$ where $A = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}$, we first need to compute A^2:

$$A^2 = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}$$

Calculating the matrix multiplication:

$$A^2 = \begin{bmatrix} (1)(1) + (2)(-2) & (1)(2) + (2)(-5) \\ (-2)(1) + (-5)(-2) & (-2)(2) + (-5)(-5) \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix}$$

Next, substitute A^2 into the equation $\alpha A^2 + \beta A = 2I$:

$$\alpha \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix} + \beta \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Break this down into:

$$\begin{bmatrix} -3\alpha & -8\alpha \\ 8\alpha & 21\alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -2\beta & -5\beta \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Equate the corresponding elements from both sides:

$-3\alpha + \beta = 2$

$-8\alpha + 2\beta = 0$

$8\alpha - 2\beta = 0$

$21\alpha - 5\beta = 2$

From the second equation $-8\alpha + 2\beta = 0$, we solve for β:

$2\beta = 8\alpha \Rightarrow \beta = 4\alpha$

Substitute $\beta = 4\alpha$ into the first equation:

$-3\alpha + 4\alpha = 2 \Rightarrow \alpha = 2$

Using $\beta = 4\alpha$, we find:

$\beta = 4(2) = 8$

Finally, calculate $\alpha + \beta$:

$\alpha + \beta = 2 + 8 = 10$

Question51

The system of equations$x + 2y + 3z = 6, x + 3y + 5z = 9,$ $2x + 5y + az = 12$ has no solution when $a =$

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Options:

A. 5

B. 6

C. 7

D. 8

Answer: D

Solution:

To find the value of a that makes the system of equations have no solution, we need to ensure that the determinant of the coefficient matrix is zero. This will indicate that the equations are linearly dependent, leading to no unique solutions.

Given the system of equations:

$$\begin{aligned} x+2y+3z&=6,\\ x+3y+5z&=9,\\ 2x+5y+az&=12, \end{aligned}$$

we consider the coefficient matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{bmatrix}$$

Calculate the determinant of this matrix:

$$D=\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{vmatrix}$$

The determinant is calculated as follows:

$$\begin{aligned} D&=1\cdot(3a-25)-2\cdot(a-10)+3\cdot(5-6)\\ &=3a-25-2a+20-3\\ &=3a-2a-25+20-3\\ &=a-8. \end{aligned}$$

Setting the determinant $D=0$ gives:

$$a-8=0,$$

which simplifies to:

$$a=8.$$

Therefore, the system of equations has no solution when $a=8$.

Question52

If α,β,γ are the roots of $\begin{bmatrix} 1 & -x & -2 \\ -2 & 4 & -x \\ -2 & 1 & -x \end{bmatrix}=0$, then $\alpha\beta+\beta\gamma+\gamma\alpha=$

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Options:

A. 6

B. 8

C. 0

D. -4

Answer: C

Solution:

We have.

$$\begin{vmatrix} 1-x & -2 & 1 \\ -2 & 4-x & -2 \\ 1 & -2 & 1-x \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 + R_2 + R_y$ we get

$$\Rightarrow \begin{vmatrix} -x & -x & -x \\ -2 & 4-x & -2 \\ 1 & -2 & 1-x \end{vmatrix} = 0$$

$$\Rightarrow -x \begin{vmatrix} 1 & 1 & 1 \\ -2 & 4-x & -2 \\ 1 & -2 & 1-x \end{vmatrix} = 0$$

$C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we get

$$\Rightarrow -x \begin{vmatrix} 1 & 0 & 0 \\ -2 & 6-x & 0 \\ 1 & -3 & -x \end{vmatrix} = 0$$

$$\Rightarrow \quad (-x)(-x)(6-x) = 0$$

$$x' - 6x^2 = 0$$

Sum of product of zero

$$= \frac{\text{Coefficient of} \quad x}{\text{Cocflicient of} \quad x^3} = 0$$

Hence, $\alpha\beta + \beta y + p^2 = 0$

Question53

If the determinant of a 3rd order matrix A is K, then the sum of the determinants of the matrices A^4 and $(A - A^4)$ is

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Options:

A. 2K

B. 0

C. K^2

D. K

Answer: C

Solution:

We are given that the determinant of a 3rd order matrix A is K. Hence, $|A| = K$.

For a matrix A, the determinant properties used here are:

The determinant of the transpose of a matrix is the same as the determinant of the matrix itself. So, $|A^T| = |A| = K$.

The determinant of the product of a matrix with its transpose is $|AA^T| = |A||A^T| = K \times K = K^2$.

Now, we are asked to find the sum of the determinants of matrices A^4 and $(A - A^4)$.

Since the determinant of a product is the product of the determinants and given $|A| = K$, we have:

$|A^4| = |A|^4 = K^4$.

To find $|A - A^4|$, note that:

By matrix properties, $|(A - A^4)| =$ simplify or evaluate separately.

Given the expresion $|(AA^T) + (A - A^T)|$ in the context, we simplify:

$|AA^T| + |(A - A^4)| = K^2 + |A| - |A^T| = K^2 + K - K = K^2$.

Thus the result for the sum of these determinants aligns with the property calculations stated, giving us the simplified determinant result of K^2.

Question54

While solving a system of linear equations $AX = B$ using Cramer's rule with the usual notation if

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ -1 & 1 & 5 \end{vmatrix}, \Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 4 & -1 & 2 \\ 11 & 1 & 5 \end{vmatrix} \text{ and } X = \begin{bmatrix} \alpha \\ 2 \\ \beta \end{bmatrix}, \text{ then } \alpha^2 + \beta^2 =$$

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Options:

A. 9

B. 13

C. 5

D. 25

Answer: C

Solution:

We have,

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ -1 & 1 & 5 \end{vmatrix}$$
$$= 1(-5-2) - 1(10+2) + 1(2-1)$$
$$= -7 - 12 + 1$$
$$= -18 \neq 0$$
$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 4 & -1 & 2 \\ 11 & 1 & 5 \end{vmatrix}$$
$$= 5(-5-2) - 1(20-22) + 1(4+11)$$
$$= -35 + 2 + 15$$
$$= -18$$
$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 2 & -1 & 4 \\ -1 & 1 & 11 \end{vmatrix}$$
$$= 1(-11-4) - 1(22+4) + 5(2-1)$$
$$= -15 - 26 + 5 = -36$$
$$\therefore \quad \alpha = \frac{\Delta_1}{\Delta} = \frac{-18}{-18} = 1,$$
$$\therefore \quad \beta = \frac{\Delta_3}{\Delta} = \frac{-36}{-18} = 2$$

Hence, $\alpha^2 + \beta^2 = 1^2 + 2^2 = 5$.

Question55

If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then AA^T is a

Options:

A. symmetric matrix

B. skew-symmetric matrix

C. singular matrix

D. inverse of A

Answer: A

Solution:

Here,

$$A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$$

$$A^T=\begin{bmatrix}3 & 2 & 0\\ -3 & -3 & -1\\ 4 & 4 & 1\end{bmatrix}$$

$$AA^T=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}3 & 2 & 0\\ -3 & -3 & -1\\ 4 & 4 & 1\end{bmatrix}$$

$$AA^T=\begin{bmatrix}9+9+16 & 6+9+16 & 0+3+4\\ 6+9+16 & 4+9+16 & 0+3+4\\ 0+3+4 & 0+3+4 & 0+1+1\end{bmatrix}$$

$$AA^T=\begin{bmatrix}34 & 31 & 7\\ 31 & 29 & 7\\ 7 & 7 & 2\end{bmatrix}$$

$$\left(AA^T\right)^T=\begin{bmatrix}34 & 31 & 7\\ 31 & 29 & 7\\ 7 & 7 & 2\end{bmatrix}$$

So, AA^T is a symmetric matrix.

Question56

If $AX=D$ represents the system of simultaneous linear equations $x+y+z=6, 5x-y+2z=3$ and $2x+y-z=-5$, then (Adj A) $D=$

Options:

A. $\begin{bmatrix} 8 \\ -16 \\ 40 \end{bmatrix}$

B. $\begin{bmatrix} 32 \\ 64 \\ -160 \end{bmatrix}$

C. $\begin{bmatrix} -16 \\ 32 \\ 80 \end{bmatrix}$

D. $\begin{bmatrix} 12 \\ 24 \\ 60 \end{bmatrix}$

Answer: C

Solution:

The given system of equation is $5x - y + 2z = 3, x + y + z = 6$ and $2x + y - z = -5$ In matrix form, if may be represented as where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } D = \begin{bmatrix} 6 \\ 3 \\ -5 \end{bmatrix}$$

Thus, A is a non-singular matrix, so A^{-1} exist.

$$C_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = -1$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 2 & -1 \end{vmatrix} = 9$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 5 & -1 \\ 2 & 1 \end{vmatrix} = 7$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 2$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} = 3$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix} = 3$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 5 & -1 \end{vmatrix} = -6$$

$$\text{Then, adj}(A)=\begin{bmatrix}-1 & 9 & 7\\ 2 & -3 & 1\\ 3 & 3 & -6\end{bmatrix}^T=\begin{bmatrix}-1 & 2 & 3\\ 9 & -3 & 3\\ 7 & 1 & -6\end{bmatrix}$$

$$(\text{adj } A)\cdot D=\begin{bmatrix}-1 & 2 & 3\\ 9 & -3 & 3\\ 7 & 1 & -6\end{bmatrix}\begin{bmatrix}6\\ 3\\ -5\end{bmatrix}$$

$$=\begin{bmatrix}-6+6-15\\ 54-9-15\\ 42+3+30\end{bmatrix}=\begin{bmatrix}-15\\ 30\\ 75\end{bmatrix}$$

Question57

If $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$**, then** $\det\left(A^6+B^6\right)=$

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Options:

A. -68

B.

- 212

C. 665

D. 720

Answer: B

Solution:

$A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$

$\Rightarrow A^2=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

$\Rightarrow A^2=\begin{bmatrix}1+0 & 0+0\\ 2+2 & 0+1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}$

$\Rightarrow A^3=\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} \quad [A^2\cdot A=A^3]$

$\Rightarrow A^3=\begin{bmatrix}1+0 & 0+0\\ 4+2 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 6 & 1\end{bmatrix}$

Similarly, $A^6=\begin{bmatrix}1 & 0\\ 12 & 1\end{bmatrix} \quad [A^n=A_{21}=n\times 2]$

Now, $B=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$

$\Rightarrow B^2=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$

$\Rightarrow B^2=\begin{bmatrix}1+0 & 3+3\\ 0+0 & 0+1\end{bmatrix}=\begin{bmatrix}1 & 6\\ 0 & 1\end{bmatrix}$

$\Rightarrow B^3=\begin{bmatrix}1 & 6\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 9\\ 0 & 1\end{bmatrix}$

Similarly, $B^6=\begin{bmatrix}1 & 18\\ 0 & 1\end{bmatrix} \quad [B^n=B_{12}=n\times 3]$

$\therefore \quad (A^6+B^6)=\begin{bmatrix}1 & 0\\ 12 & 1\end{bmatrix}+\begin{bmatrix}1 & 18\\ 0 & 1\end{bmatrix}$

$A^6+B^6=\begin{bmatrix}2 & 18\\ 12 & 2\end{bmatrix}$

Det $(A^6+B^6)=4-216=-212$

Question58

Let $G(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$. If $x+y=0$ then $G(x)G(y)=$

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Options:

A. null Matrix

B. skew-symmetric Matrix

C. identity Matrix

D. symmetric Matrix

Answer: C

Solution:

Here, $G(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

$G(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

So, $x+y=0 \Rightarrow y=(-x)$

Now, $G(-x)=\begin{bmatrix}\cos(-x) & -\sin(-x) & 0\\ \sin(-x) & \cos(-x) & 0\\ 0 & 0 & 1\end{bmatrix}$

$$G(x)\cdot G(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

Question59

If $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$**, then** $\left(A^T\right)^2+(12A)^T=$

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Options:

A. $5\begin{bmatrix}8 & 12\\ -9 & 5\end{bmatrix}$

B. $5\begin{bmatrix}8 & -9\\ -12 & 5\end{bmatrix}$

C. $\begin{bmatrix}40 & -45\\ 60 & 25\end{bmatrix}$

D. $\begin{bmatrix} 40 & -60 \\ -45 & 25 \end{bmatrix}$

Answer: D

Solution:

Given, $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$$\begin{aligned} &\left(A^T\right)^2 + (12A)^T \\ &= \begin{bmatrix} 2 & -4 \\ -3 & 1 \end{bmatrix}^2 + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}^T \\ &= \begin{bmatrix} 16 & -12 \\ -9 & 13 \end{bmatrix} + \begin{bmatrix} 24 & -48 \\ -36 & 12 \end{bmatrix} \\ &= \begin{bmatrix} 16+24 & -12-48 \\ -9-36 & 13+12 \end{bmatrix} = \begin{bmatrix} 40 & -60 \\ -45 & 25 \end{bmatrix} \end{aligned}$$

Question60

If a, b, c are respectively the 5 th, 8 th, 13 th terms of an arithmetic progression, then $\begin{vmatrix} a & 5 & 1 \\ b & 8 & 1 \\ c & 13 & 1 \end{vmatrix} =$

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Options:

A. 0

B. 1

C. abc

D. 520

Answer: A

Solution:

Given, a, b and c are respectively 5th, 8th, and 13th terms of arithmetic progression.

$\therefore a = P + (5-1)d$
$\Rightarrow a = P + 4d$
$b = P + (8-1)d$
$\Rightarrow \quad b = P + 7d$
$c = P + (13-1)d$
$\Rightarrow c = P + 12d$

$$\therefore \quad \begin{vmatrix} a & 5 & 1 \\ b & 8 & 1 \\ c & 13 & 1 \end{vmatrix} = \begin{vmatrix} P+4d & 5 & 1 \\ P+7d & 8 & 1 \\ P+12d & 13 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} P & 5 & 1 \\ P & 8 & 1 \\ P & 13 & 1 \end{vmatrix} + \begin{vmatrix} 4d & 5 & 1 \\ 7d & 8 & 1 \\ 12d & 13 & 1 \end{vmatrix}$$

$$= P \begin{vmatrix} 1 & 5 & 1 \\ 1 & 8 & 1 \\ 1 & 13 & 1 \end{vmatrix} + d \begin{vmatrix} 4+1 & 5 & 1 \\ 7+1 & 8 & 1 \\ 12+1 & 13 & 1 \end{vmatrix}$$

$$= P \cdot 0 + d \begin{vmatrix} 5 & 5 & 1 \\ 8 & 8 & 1 \\ 13 & 13 & 1 \end{vmatrix} = P \cdot 0 + d \cdot 0 = 0$$

Question61

If $A = \begin{bmatrix} 1 & 0 & 0 \\ a & -1 & 0 \\ b & c & 1 \end{bmatrix}$ is such that $A^2 = I$, then

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Options:

A. $b = \frac{ac}{2}$

B. $b = -\frac{ac}{2}$

C. $b = \frac{a+c}{2}$

D. $b = \sqrt{ac}$

Answer: B

Solution:

Given that,

$$A=\begin{bmatrix}1 & 0 & 0\\ a & -1 & 0\\ b & c & 1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 0 & 0\\ a & -1 & 0\\ b & c & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ a & -1 & 0\\ b & c & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ b+ca+b & 0 & 1\end{bmatrix}$$

Since, $A^2=I$ (given)

$\Rightarrow b+ca+b=0$

$\Rightarrow b=\frac{-ca}{2}$

Question62

Let $A=\begin{bmatrix}-2 & x & 1\\ x & 1 & 1\\ 2 & 3 & -1\end{bmatrix}$**. If the roots of the equation** $\det A=0$ **are** l, m **then** $l^3-m^3=$

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Options:

A. 35

B. -35

C. 19

D. -19

Answer: C

Solution:

Given,

$$A=\begin{bmatrix}-2 & x & 1\\ x & 1 & 1\\ 2 & 3 & -1\end{bmatrix}$$

Given, $\det A=\begin{vmatrix}-2 & x & 1\\ x & 1 & 1\\ 2 & 3 & -1\end{vmatrix}=0$

$\Rightarrow \quad -2(-1-3)+x(2+x)+1(3x-2)=0$
$\Rightarrow \quad 8+2x+x^2+3x-2=0$
$\Rightarrow \quad x^2+5x+6=0 \Rightarrow (x+2)(x+3)=0$
$\Rightarrow \quad x=-2 \text{ and } -3$
$\therefore \quad l=-2 \text{ and } m=-3$
$\quad l^3-m^3=(-2)^3-(-3)^3=-8+27=19$

Question63

For $i=1,2,3$ and $j=1,2$3If $a_i^2+b_i^2+c_i^2=1, a_ia_j+b_ib_j+c_ic_j=0, \forall i \neq j$and $A=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, then $\det\left(AA^T\right)=$

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Options:

A. 0

B. 1

C. -1

D. 3

Answer: B

Solution:

Given, $a_i^2+b_i^2+c_i^2=1$

$$\Rightarrow \quad a_1^2+b_1^2+c_1^2=1$$
$$a_2^2+b_2^2+c_2^2=1$$
$$a_3^2+b_3^2+c_3^2=1$$

Given, $a_ia_j+b_ib_j+c_ic_j=0 \quad [i,j=1,2,3, i\neq j]$

Let $\mathbf{p}=a_1\mathbf{i}+b_1\mathbf{j}+c_1\mathbf{k}$
$\mathbf{q}=a_2\mathbf{i}+b_2\mathbf{j}+c_2\mathbf{k}$
$\mathbf{r}=a_3\mathbf{i}+b_3\mathbf{j}+c_3\mathbf{k}$
$|\mathbf{p}|=|\mathbf{q}|=|\mathbf{r}|=1$
$\mathbf{p}\cdot\mathbf{q}=(a_1\mathbf{i}+b_1\mathbf{j}+c_1\mathbf{k})\cdot(a_2\mathbf{i}+b_2\mathbf{j}+c_3\mathbf{k})$
$=a_1a_2+b_1b_2+c_1c_2=0$
$\Rightarrow \mathbf{p}\perp\mathbf{q}$

Similarly, $\mathbf{q} \perp \mathbf{r}, \mathbf{r} \perp \mathbf{p}$

$$\text{Now, } A = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = [\mathbf{p} \quad \mathbf{q} \quad \mathbf{r}] = 1$$

$$\Rightarrow \det(AA^T) = 1$$

Question64

If $A = \frac{1}{7}\begin{bmatrix} 3 & -2 & 6 \\ -6 & -3 & 2 \\ -2 & 6 & 3 \end{bmatrix}$, then

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Options:

A. $A^{-1} = A$

B. $A^{-1} = A^T$

C. A^{-1} does not exist

D. $A^{-1} = -A$

Answer: B

Solution:

$$A = \frac{1}{7}\begin{bmatrix} 3 & -2 & 6 \\ -6 & -3 & 2 \\ -2 & 6 & 3 \end{bmatrix}$$

$$|A| = \frac{1}{7^3}\begin{bmatrix} 3 & -2 & 6 \\ -6 & -3 & 2 \\ -2 & 6 & 3 \end{bmatrix} \quad [\because \det(kA) = k^n \det A]$$

$$= \frac{1}{7^3}[3(-9-12) + 2(-18+4) + 6(-36-6)]$$

$$= \frac{1}{7^3}[-63-28-252]$$

$$= \frac{-343}{7^3} = -1 \neq -0$$

$$\text{Adj}(A) = \frac{1}{7^2}\begin{bmatrix} -21 & 14 & -42 \\ 42 & 21 & -14 \\ 14 & -42 & -21 \end{bmatrix}$$

$[\because \operatorname{adj}(kA) = k^{n-1} \operatorname{adj} A]$

$$= \frac{1}{7^2}\begin{bmatrix} -21 & 42 & 14 \\ 14 & 21 & -42 \\ -42 & -14 & -21 \end{bmatrix}$$

Now, $A^{-1} = \frac{\operatorname{adj}(A)}{|A|} = \frac{-1}{7^2}\begin{bmatrix} -21 & 42 & 14 \\ 14 & 21 & -42 \\ -42 & -14 & -21 \end{bmatrix}$

$$= \frac{1}{7}\begin{bmatrix} 3 & -6 & -2 \\ -2 & -3 & 6 \\ 6 & 2 & 3 \end{bmatrix}$$

$A^{-1} = A^T$

Question65

If $A = \begin{bmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{bmatrix}$ **and** $\det\left(A^{10}\right) = 1024$**, then** $\alpha =$

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Options:

A. -2

B. -1

C. -3

D. 0

Answer: C

Solution:

Step 1: Given matrix A

The matrix A is given as:

$$A = \begin{bmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{bmatrix}$$

Step 2: Find the determinant of A

The determinant of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is calculated by:

$$\det(A) = ad - bc$$

For matrix A, we have:

$$a = \alpha^2, \quad b = 5, \quad c = 5, \quad d = -\alpha$$

So,

$$\det(A) = (\alpha^2)(-\alpha) - (5)(5) = -\alpha^3 - 25$$

Thus, the determinant of A is:

$$\det(A) = -\alpha^3 - 25$$

Step 3: Use the condition $\det(A^{10}) = 1024$

Now, we are given that $\det(A^{10}) = 1024$. The property of determinants tells us that:

$$\det(A^n) = (\det(A))^n$$

Thus:

$$\det(A^{10}) = (\det(A))^{10}$$

We are told that:

$$(\det(A))^{10} = 1024$$

Since $1024 = 2^{10}$, we have:

$$(\det(A))^{10} = 2^{10}$$

Taking the 10th root of both sides:

$$\det(A) = 2$$

Step 4: Solve for α

From earlier, we found that:

$$\det(A) = -\alpha^3 - 25$$

We know that $\det(A) = 2$, so:

$$-\alpha^3 - 25 = 2$$

Solving for α^3:

$$-\alpha^3 = 27$$

$$\alpha^3 = -27$$

Taking the cube root of both sides:

$$\alpha = -3$$

Final Answer:

$$\boxed{-3}$$

Question66

Let $A = \begin{bmatrix} 5 & \sin^2\theta & \cos^2\theta \\ -\sin^2\theta & -5 & 1 \\ \cos^2\theta & 1 & 5 \end{bmatrix}$**. Then, maximum value of** $\det(A)$ **is**

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Options:

A. -125

B. 200

C. $-\frac{255}{2}$

D. 145

Answer: A

Solution:

$$A=\begin{bmatrix}5 & \sin^2\theta & \cos^2\theta \\ -\sin^2\theta & -5 & 1 \\ \cos^2\theta & 1 & 5\end{bmatrix}$$

$$\begin{aligned}|A| &= 5(-25-1)-\sin^2\theta\left(-5\sin^2\theta-\cos^2\theta\right) \\ &+\cos^2\theta\left(-\sin^2\theta+5\cos^2\theta\right) \\ &= -130-\sin^2\theta\left[-5\left(1-\cos^2\theta\right)-\cos^2\theta\right] \\ &+\cos^2\theta\left[-\sin^2\theta+5\left(1-\sin^2\theta\right)\right] \\ &= -130-\sin^2\theta\left(-5+4\cos^2\theta\right)+\cos^2\theta\left(-6\sin^2\theta+5\right) \\ &= -130+5\sin^2\theta-4\sin^2\theta\cos^2\theta \\ &-6\sin^2\theta\cos^2\theta+5\cos^2\theta \\ &= -130+5-10\sin^2\theta\cos^2\theta \\ |A| &= -125-10\sin^2\theta\cos^2\theta \\ \therefore \quad |A|_{\max} &= -125\end{aligned}$$

Question67

If $\frac{x^4+24x^2+28}{(x^2+1)^3}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{Ex+F}{(x^2+1)^3}$, then the value of $A+B+C+D+E+F=$

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Options:

A. 21

B. 22

C. 28

D. 29

Answer: C

Solution:

$$\frac{x^4+24x^2+28}{(x^2+1)^3}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{Ex+F}{(x^2+1)^3}$$

Let $x^2+1=k$

$\Rightarrow \quad x^2=k-1$

Substituting $x^2=k-1$ in LHS, we get

$$\frac{(k-1)^2+24(k-1)+28}{k^3}=\frac{k^2+22k+5}{k^3}$$
$$=\frac{1}{k}+\frac{22}{k^2}+\frac{5}{k^3}$$
$$=\frac{1}{(x^2+1)}+\frac{22}{(x^2+1)^2}+\frac{5}{(x^2+1)^3}$$

On comparing with RHS, we get

$Ax+B=1, Cx+D=22$ and $Ex+F=5$
At $x=1$,
$A+B=1, C+D=22$ and $E+F=5$
$\therefore A+B+C+D+E+F=1+22+5=28$

Question68

If $k \in R$ and $\det A=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=k$, then $\det B=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2+2a_1 & b_2+2b_1 & c_2+2c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is equal to

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Options:

A. 0

B. 2k

C. k

D. k^2

Answer: C

Solution:

$$\det A=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=k \text{ (given)}$$

$$\text{Then, } \det B = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 + 2a_1 & b_2 + 2b_1 & c_2 + 2c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ 2a_1 & 2b_1 & 2c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= k + 0$$

$$\Rightarrow \det B = k$$

Question69

If $A = \begin{bmatrix} \sqrt{2020} & \sqrt{2021} & \sqrt{2021} & \sqrt{2023} \\ \sqrt{4040} & \sqrt{4042} & \sqrt{4044} & \sqrt{4046} \\ \sqrt{6060} & \sqrt{6063} & \sqrt{6066} & \sqrt{6069} \\ \sqrt{8080} & \sqrt{8084} & \sqrt{8088} & \sqrt{8092} \end{bmatrix}$, then the rank of A is

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Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

$$A=\begin{bmatrix}\sqrt{2020} & \sqrt{2022} & \sqrt{2022} & \sqrt{2023}\\ \sqrt{4040} & \sqrt{4042} & \sqrt{4044} & \sqrt{4046}\\ \sqrt{6060} & \sqrt{6063} & \sqrt{6066} & \sqrt{6069}\\ \sqrt{8080} & \sqrt{8084} & \sqrt{8088} & \sqrt{8092}\end{bmatrix}$$
$$=\begin{bmatrix}\sqrt{2020} & \sqrt{2021} & \sqrt{2022} & \sqrt{2023}\\ \sqrt{2}\cdot\sqrt{2020} & \sqrt{2}\sqrt{2021} & \sqrt{4044} & \sqrt{4046}\\ \sqrt{3}\sqrt{2020} & \sqrt{3}\sqrt{2021} & \sqrt{6066} & \sqrt{6069}\\ \sqrt{4}\sqrt{2020} & \sqrt{4}\sqrt{2021} & \sqrt{8088} & \sqrt{8092}\end{bmatrix}$$
$R_2 \to R_2-\sqrt{2}R_1, R_3 \to R_3-\sqrt{3}R_1, R_4 \to R_4-\sqrt{4}R_1$
$$A\sim\begin{bmatrix}\sqrt{2020} & \sqrt{2021} & \sqrt{2022} & \sqrt{2023}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$\because$ A has one non-zero row

$\therefore$ Rank (A) = 1

Option (a) is correct.

Question70

If $\begin{vmatrix} x & x^2 & 1+x^3\\ y & y^2 & 1+y^3\\ z & z^2 & 1+z^3\end{vmatrix}=0$ **and** x, y **and** z **are all distinct, then** xyz **is equal to**

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Options:

A. -1

B. 1

C. 0

D. 3

Answer: A

Solution:

The given equation is:

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

We can rewrite the third column as (1) + (the cube term), so this is the same as:

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

Let's call the first determinant as A and the second as B.

Step 1: Find the value of $|A|$

Let: $A = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}$

Subtract first row from second and third, $R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$ so, $\begin{vmatrix} x & x^2 & 1 \\ y-x & y^2-x^2 & 0 \\ z-x & z^2-x^2 & 0 \end{vmatrix}$

Now, since last column for R_2 and R_3 is 0, expand along third column:

$|A| = (y-x)(z^2-x^2) - (z-x)(y^2-x^2)$

$= (y-x)(z-x)(z+x) - (z-x)(y-x)(y+x)$

$= (y-x)(z-x)(z+x-y-x)$

$= (y-x)(z-x)(z-y)$

Step 2: Find the value of $|B|$

$B = \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$

Factor x, y, z from each row: $= xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

Now, do the same row operation as before: $R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$

So determinant becomes: $xyz \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$

Expand this along the first column:

$|B| = xyz\left\{(y-x)(z^2-x^2) - (z-x)(y^2-x^2)\right\}$

$= xyz(y-x)(z-x)(z-y)$

Step 3: Use the original equation

We have: $|A| + |B| = 0$ So, $(y-x)(z-x)(z-y) + xyz(y-x)(z-x)(z-y) = 0$

Factor out $(y-x)(z-x)(z-y)$: $(y-x)(z-x)(z-y)(1+xyz) = 0$

Because x, y, z are all different, $(y-x)(z-x)(z-y) \neq 0$.So,$1 + xyz = 0$

Therefore,$xyz = -1$

Question71

Let A be a $n \times n$ matrix such that A is upper-triangular. Then, $adj(A)$ is equal to

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Options:

A. lower triangular matrix

B. upper triangular matrix

C. diagonal matrix

D. scalar matrix

Answer: B

Solution:

Let $n = 2$ (say) and $A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$, Then adj $A = \begin{bmatrix} c & -b \\ 0 & a \end{bmatrix}$ which is a upper triangular matrix. Then, for any $n \times n$ matrix A, adj A is also upper triangular matrix.

Question72

If $f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$, then the ratio $f''(x) : f'(x)$ is equal to

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Options:

A. $2 : x$

B. $x^2 : x$

C. $3x : 2$

D. $6 : x$

Answer: A

Solution:

$$f(x)=\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$$

Expand along column 1

$$\begin{aligned} f(x) &= x\left(12x^2-6x^2\right)-\left(6x^3-2x^3\right) \\ &= x\left(6x^2\right)-4x^3=6x^3-4x^3=2x^3 \end{aligned}$$

$$\therefore \quad f(x)=2x^3 \Rightarrow f'(x)=6x^2$$

$$f''(x)=12x$$

Then, $f''(x) : f'(x)=12x : 6x^2=2 : x$

Question73

The trace of the matrix $A=\begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$ is

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Options:

A. 17

B. 25

C. 3

D. 12

Answer: A

Solution:

Trace of A = Sum of diagonal elements of A

Given, $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$

$= 1 + 7 + 9 = 17$

Question74

If A, B and C are the angles of a triangle, then the system of equations $-x + y \cos C + z \cos B = 0, x \cos C - y + z \cos A = 0$ and $x \cos B + y \cos A - z = 0$

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Options:

A. Only zero solution

B. A non-zero solution for all ΔABC

C. Only zero solution but for certain values of A, Band C

D. A non-zero solution if ΔABC is an equilateraltriangle and not for all triangles.

Answer: B

Solution:

$$-x + y \cos C + z \cos B = 0$$
$$x \cos C - y + z \cos A = 0$$
$$x \cos B + y \cos A - Z = 0$$
$$\Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$$
$$= -1\left(1 - \cos^2 A\right) + \cos C(\cos A \cos B + \cos C)$$
$$+ \cos B(\cos A \cos C + \cos B)$$

$$= -1 + \cos^2 A + 2 \cos A \cos B \cos C$$
$$+ \cos^2 C + \cos^2 B$$
$$= -\sin^2 A + \cos^2 B + \cos^2 C + 2 \cos A \cos B \cos C$$

Clearly, $\Delta > 0$.

$\Rightarrow$ System of equations has a non-zero solution.

Question75

If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$**, then**

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Options:

A. $a = 1, b = 1$

B. $a = \sin 2\theta$ and $b = \cos 2\theta$

C. $a = \cos 2\theta$ and $b = \sin 2\theta$

D. $a = 0$ and $b = 0$

Answer: C

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\operatorname{adj}(A)}{|A|}$$

$$= \left(\frac{1}{\sec^2\theta}\right)\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$\text{So, } \frac{1}{\sec^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2\theta}\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & -\tan^2\theta + 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \quad a = \cos 2\theta, b = \sin 2\theta$$

Question76

What is the value of $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$ **?**

Options:

A. $a^3+b^3+c^3+3abc$

B. $a^3+b^3+c^3-3abc$

C. $a^3+b^3+c^3-6abc$

D. $a^3+b^3+c^3+6abc$

Answer: B

Solution:

$$\begin{aligned}
&\begin{bmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{bmatrix} \quad [R_1 \rightarrow R_1+R_3] \\
&=(a+b+c)\begin{bmatrix} 1 & 1 & 1 \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{bmatrix} \\
&=(a+b+c)\begin{bmatrix} 1 & 1 & 1 \\ a+c & b+a & b+c \\ b+c & c+a & a+b \end{bmatrix} \quad [R_2 \rightarrow R_2+R_3]
\end{aligned}$$

$$\begin{aligned}
&C_2 \rightarrow C_2-C_1 \text{ and } C_3 \rightarrow C_3-C_1 \\
&\quad=(a+b+c)\begin{bmatrix} 1 & 0 & 0 \\ a+c & b-c & b-a \\ b+c & a-b & a-c \end{bmatrix} \\
&\quad=(a+b+c)[(b-c)(a-c)-(b-a)(a-b)] \\
&\quad=(a+b+c)\left(ab-bc-ca+c^2-ba+b^2+a^2-ab\right) \\
&\quad=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right) \\
&\quad=a^3+b^3+c^3-3abc
\end{aligned}$$

Question77

The value of $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ **is**

Options:

A. abc

B. $(a+b)(b+c)(c+a)$

C. $4abc$

D. $(a-b)(b-c)(c-a)$

Answer: C

Solution:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$
$$= (b+c)[(c+a)(a+b)-bc] - a\left(ab+b^2-bc\right)$$
$$= (b+c)\left(ac+bc+a^2+ab-bc\right) \quad +a\left(bc-c^2-ac\right)$$
$$-a^2b-ab^2+abc+abc-ac^2-a^2c$$
$$= abc+a^2b+ab^2+ac^2+a^2c+abc-a^2b-ab^2+abc+abc-ac^2-a^2c$$
$$= 4abc$$

Option (c) is correct.

Question78

Let A, B, C, D be square real matrices such that $C^T = DAB, D^{\mathrm{T}} = ABC$ and $S = ABCD$, then S^2 is equal to

Options:

A. S

B. BCD

C. S^T

D. $\left(S^T\right)^2 = \left(S^2\right)^T$

Answer: D

Solution:

$D^T = ABC$

and $S = ABCD$

$S^2 = (ABCD)(ABCD) = (D^T . D)(D^T D)$

$= (D^T D)^2$ ..... (i)

$\because S = ABCD = D^T D$

$S^T = (D^T D)^T = D^T D$

$\therefore$ From Eq. (i), we get

$S^2 = (S^T)^2$

$\therefore S^2 = (S^T)^2 = (S^2)^T$

Question79

$A = \begin{bmatrix} a^2 & 15 & 31 \\ 12 & b^2 & 41 \\ 35 & 61 & c^2 \end{bmatrix}$ and $B = \begin{bmatrix} 2a & 3 & 5 \\ 2 & 2b & 8 \\ 1 & 4 & 2c-3 \end{bmatrix}$ are two matrices such that the sum of the principal diagonal elements of both A and B are equal, then the product of the principal diagonal elements of B is

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Options:

A. 4

B. 0

C. -4

D. -12

Answer: C

Solution:

$$A = \begin{bmatrix} a^2 & 15 & 31 \\ 12 & b^2 & 41 \\ 35 & 61 & c^2 \end{bmatrix},$$
$$B = \begin{bmatrix} 2a & 3 & 5 \\ 2 & 2b & 8 \\ 1 & 4 & 2c-3 \end{bmatrix}$$

Given, trace of A = trace of B

$$\Rightarrow a^2 + b^2 + c^2 = 2a + 2b + 2c - 3$$
$$\Rightarrow \left(a^2 - 2a + 1\right) + \left(b^2 - 2b + 1\right) + \left(c^2 - 2c + 1\right) = 0$$
$$\Rightarrow (a-1)^2 + (b-1)^2 + (c-1)^2 = 0$$
$$\Rightarrow a = 1, b = 1, c = 1$$

Then product of diagonal elements of B

$$= (2a)(2b)(2c-3) = (2)(2)(2-3) = -4$$

Question80

Let a, b and c be such that $b + c \neq 0$ and

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n-1}b & (-1)^{n}c \end{vmatrix} = 0,$$

then the value of n is

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Options:

A. zero

B. any even integer

C. any odd integer

D. any integer

Answer: C

Solution:

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

$$+\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n-1}b & (-1)^n c \end{vmatrix} = 0$$

$\because |A| =| A^T]$

$$\Rightarrow \begin{vmatrix} a & -b & c \\ a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \end{vmatrix}$$
$$+(-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix} = 0$$

$R_2 \leftrightarrow R_3$ and $R_3 \leftrightarrow R_1$ in 2nd determinant

$$\Rightarrow \begin{vmatrix} a & -b & c \\ a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \end{vmatrix}$$
$$+(-1)^n \begin{vmatrix} a & -b & c \\ a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \end{vmatrix} = 0$$

Both determinants are equal and their sum is zero when n is an odd integer.

Question81

The equation whose roots are the values of the equation $\begin{vmatrix} 1 & -3 & 1 \\ 1 & 6 & 4 \\ 1 & 3x & x^2 \end{vmatrix} = 0$ is

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Options:

A. $x^2 + x + 2 = 0$

B. $x^2 + x - 2 = 0$

C. $x^2 + 2x + 2 = 0$

D. $x^2 - x - 2 = 0$

Answer: D

Solution:

$$\begin{vmatrix} 1 & -3 & 1 \\ 1 & 6 & 4 \\ 1 & 3x & x^2 \end{vmatrix} = 0$$
$$\Rightarrow 1\left(6x^2 - 12x\right) + 3\left(x^2 - 4\right) + 1(3x - 6) = 0$$
$$\Rightarrow 6x^2 - 12x + 3x^2 - 12 + 3x - 6 = 0$$
$$\Rightarrow 9x^2 - 9x - 18 = 0$$
$$\Rightarrow x^2 - x - 2 = 0$$

Question82

Let a and b be non-zero real numbers such that $ab = 5/2$ and given $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ and $AA^T = 20I$ (l is unit matrix), then the equation whose roots are a and b is

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Options:

A. $x^2 \mp 10x + 5 = 0$

B. $2x^2 \pm 10x + 5 = 0$

C. $x^2 - 5x + \frac{5}{2} = 0$

D. $x^2 - 25x + \frac{5}{2} = 0$

Answer: B

Solution:

$A=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

Then, $A^T=\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$

$\because \quad AA^T=20I$

$\Rightarrow \quad \begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ -b & a \end{bmatrix}=20\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}=\begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$

$\therefore \quad a^2+b^2=20$

$\Rightarrow \quad (a+b)^2-2ab=20$

$\Rightarrow \quad (a+b)^2=20+2\times\frac{5}{2} \quad \left[\because ab=\frac{5}{2}\right]$

or $\quad (a+b)^2=25$

or $\quad a+b=\pm 5$

Equation whose roots are a and b is given by$x^2+(a+b)x+ab=0$

or $\quad x^2\pm 5x+\frac{5}{2}=0$ or $2x^2\pm 10x+5=0$

Question83

If $A=\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, 10B=\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ and $B=A^{-1}$, then the value of α is

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Options:

A. 2

B. 0

C. 5

D. 4

Answer: C

Solution:

(c) $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$

$\therefore |A|=1(1+3)+1(2+3)+1(2-1)=10 \quad \ldots(\text{i})$

and $10B=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{bmatrix}$

$\therefore |10B|=4(2\alpha)-2(-15-\alpha)+2(10)$

$\Rightarrow 10^3|B|=10\alpha+50 \quad [\because |kA|=k^n|A|]\ldots(\text{ii})$

$\because B=A^{-1}$

$\Rightarrow |B|=\left|A^{-1}\right|=|A|^{-1}$

$\Rightarrow |B|=\frac{1}{|A|}=\frac{1}{10} \quad$ [[rom Eq. (i)

[from Eq. (i)]From Eq. (ii), we get

$1000\ .\ \frac{1}{10}=10\alpha+50$

or $\alpha=\frac{50}{10}=5$

Question84

The rank of the matrix $\begin{bmatrix}4 & 2 & (1-x)\\ 5 & k & 1\\ 6 & 3 & (1+x)\end{bmatrix}$ is 1 , then,

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Options:

A. $k=\frac{5}{2}, x=\frac{1}{5}$

B. $k=\frac{5}{2}, x\neq\frac{1}{5}$

C. $k=\frac{1}{5}, x=\frac{5}{2}$

D. $k\neq\frac{5}{2}, x=\frac{1}{5}$

Answer: A

Solution:

Let $A=\begin{bmatrix}4 & 2 & 1-x\\ 5 & k & 1\\ 6 & 3 & 1+x\end{bmatrix}$

Given that, rank of matrix A is 1 .

$\Rightarrow$ All the minors of order 2 are zero.

$\therefore M_{33}=0$

$\Rightarrow\left|\begin{array}{ll}4 & 2 \\ 5 & k\end{array}\right|=0$

or $4k-10=0$ or $k=\frac{5}{2}$

and $M_{11}=\left|\begin{array}{cc}k & 1 \\ 3 & 1+x\end{array}\right|=0$

or $\left|\begin{array}{cc}\frac{5}{2} & 1 \\ 3 & 1+x\end{array}\right|=0$

or $\frac{5}{2}+\frac{5}{2} x-3=0$

or $\frac{5}{2} x=\frac{1}{2}$

or $x=\frac{1}{5}$

Question85

If $a_1, a_2, \ldots . a_9$ are in GP, then $\left|\begin{array}{lll}\log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9\end{array}\right|$ is equal to

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Options:

A. $\log \left(a_1, a_2, \ldots a_n\right)$

B. 1

C. $\left(\log a_9\right)^9$

D. 0

Answer: D

Solution:

$a_1, a_2, \ldots, a_9$, are in GP.

Then, $\frac{a_2}{a_1}=\frac{a_3}{a_2}=\ldots \frac{a_9}{a_8}=r$ [common ratio]

Now, $\begin{vmatrix} \log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9 \end{vmatrix}$

$C_3 \rightarrow C_3 - C_2, C_2 \rightarrow C_2 - C_1$

$$\begin{vmatrix} \log a_1 & \log \frac{a_2}{a_1} & \log \frac{a_3}{a_2} \\ \log a_4 & \log \frac{a_5}{a_4} & \log \frac{a_6}{a_5} \\ \log a_7 & \log \frac{a_8}{a_7} & \log \frac{a_9}{a_8} \end{vmatrix}$$

or $\begin{vmatrix} \log a_1 & \log r & \log r \\ \log a_4 & \log r & \log r \\ \log a_7 & \log r & \log r \end{vmatrix} = 0$

[$\because$ C_2 and C_3 are identical]

Question86

If $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{c} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, then the value of $\begin{vmatrix} \mathbf{a}\cdot\mathbf{a} & \mathbf{a}\cdot\mathbf{b} & \mathbf{a}\cdot\mathbf{c} \\ \mathbf{b}\cdot\mathbf{a} & \mathbf{b}\cdot\mathbf{b} & \mathbf{b}\cdot\mathbf{c} \\ \mathbf{c}\cdot\mathbf{a} & \mathbf{c}\cdot\mathbf{b} & \mathbf{c}\cdot\mathbf{c} \end{vmatrix}$ is equal to

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Options:

A. 2020

B. 2025

C. 2030

D. 1849

Answer: B

Solution:

$\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}, \mathbf{c} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$
$\mathbf{a}\cdot\mathbf{a} = 4 + 1 + 9 = 14$
$\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a} = 2 + 3 - 3 = 2$

$\mathbf{a} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 6 - 1 - 6 = -1$

b. $\mathbf{b} = 1 + 9 + 1 = 11$

$\mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{b} = 3 - 3 + 2 = 2$

$\mathbf{c} \cdot \mathbf{c} = 9 + 1 + 4 = 14$

$$\text{Now, } \begin{vmatrix} \mathbf{a} \cdot \mathbf{a} & \mathbf{a} \cdot \mathbf{b} & \mathbf{a} \cdot \mathbf{c} \\ \mathbf{b} \cdot \mathbf{a} & \mathbf{b} \cdot \mathbf{b} & \mathbf{b} \cdot \mathbf{c} \\ \mathbf{c} \cdot \mathbf{a} & \mathbf{c} \cdot \mathbf{b} & \mathbf{c} \cdot \mathbf{c} \end{vmatrix} = \begin{vmatrix} 14 & 2 & -1 \\ 2 & 11 & 2 \\ -1 & 2 & 14 \end{vmatrix}$$

$= 14(154 - 4) - 2(28 + 2) - 1(4 + 11)$

$= 2100 - 60 - 15$

$= 2025$
